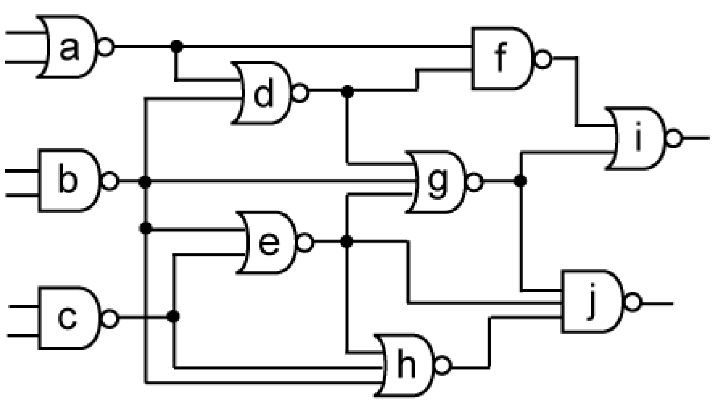
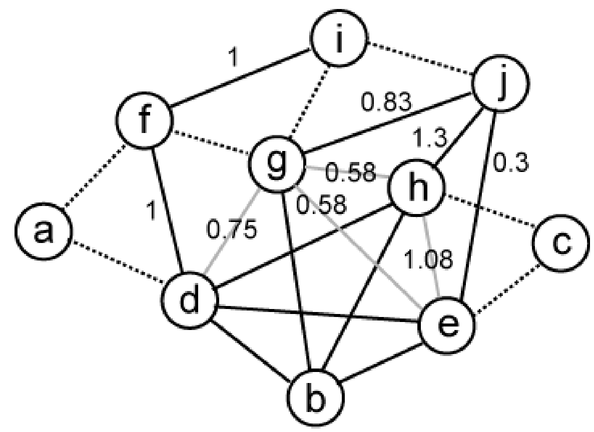
Hagen-Kahng EIG Partitioning

- Perform EIG partitioning and minimize ratio cut cost.
  - Clique-based graph model: dotted edge has weight of 0.5, and solid edge with no label has weight of 0.25.



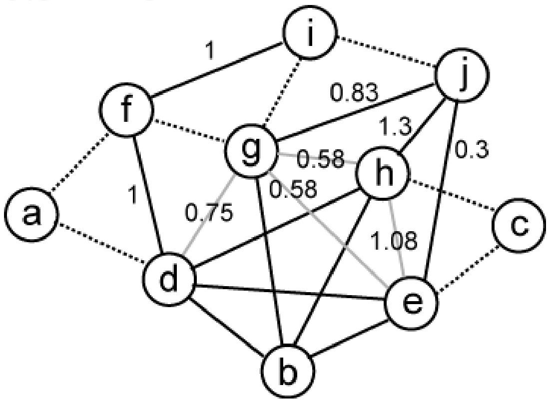
circuit



clique-based model

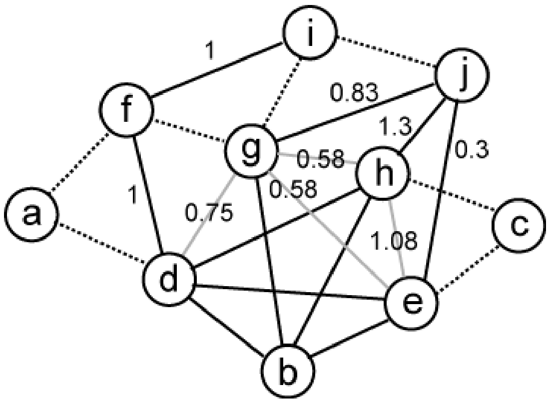
Adjacency Matrix

	a	b	c	d	e	f	g	h	i	j
a	0	0	0	0.5	0	0.5	0	0	0	0
b	0	0	0	0.25	0.25	0	0.25	0.25	0	0
c	0	0	0	0	0.5	0	0	0.5	0	0
d	0.5	0.25	0	0	0.25	1.0	0.75	0.25	0	0
e	0	0.25	0.5	0.25	0	0	0.58	1.08	0	0.33
f	0.5	0	0	1.0	0	0	0.5	0	1.0	0
g	0	0.25	0	0.75	0.58	0.5	0	0.58	0.5	0.83
h	0	0.25	0.5	0.25	1.08	0	0.58	0	0	1.33
i	0	0	0	0	0	1.0	0.5	0	0	0.5
j	0	0	0	0	0.33	0	0.83	1.33	0.5	0



Degree Matrix

	a	b	c	d	e	f	g	h	i	j
a	1.0	0	0	0	0	0	0	0	0	0
b	0	1.0	0	0	0	0	0	0	0	0
c	0	0	1.0	0	0	0	0	0	0	0
d	0	0	0	3.0	0	0	0	0	0	0
e	0	0	0	0	2.99	0	0	0	0	0
f	0	0	0	0	0	3.0	0	0	0	0
g	0	0	0	0	0	0	3.99	0	0	0
h	0	0	0	0	0	0	0	3.99	0	0
i	0	0	0	0	0	0	0	0	2.0	0
j	0	0	0	0	0	0	0	0	0	2.99



Laplacian Matrix

- We obtain $Q = D - A$

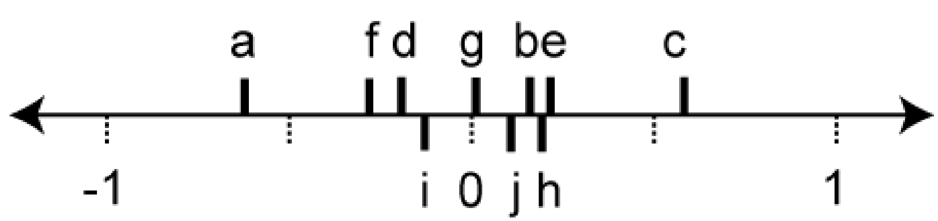
	a	b	c	d	e	f	g	h	i	j
a	1.0	0	0	-0.5	0	-0.5	0	0	0	0
b	0	1.0	0	-0.25	-0.25	0	-0.25	-0.25	0	0
c	0	0	1.0	0	-0.5	0	0	-0.5	0	0
d	-0.5	-0.25	0	3.0	-0.25	-1.0	-0.75	-0.25	0	0
e	0	-0.25	-0.5	-0.25	2.99	0	-0.58	-1.08	0	-0.33
f	-0.5	0	0	-1.0	0	3.0	-0.5	0	-1.0	0
g	0	-0.25	0	-0.75	-0.58	-0.5	3.99	-0.58	-0.5	-0.83
h	0	-0.25	-0.5	-0.25	-1.08	0	-0.58	3.99	0	-1.33
i	0	0	0	0	0	-1.0	-0.5	0	2.0	-0.5
j	0	0	0	0	-0.33	0	-0.83	-1.33	-0.5	2.99

Eigenvalue/vector Computation

The second smallest eigenvalue is 0.6281, and its eigenvector is: [-0.6346, 0.1605, 0.5711, -0.1898, 0.2254, -0.2822, 0.0038, 0.1995, -0.1641, 0.1104]^T. We observe the following:

- The squared sum of the values in the vector is 1 as shown by Hall [Hall, 1970].
- These values define a one-dimensional placement of the 10 nodes within the range of $[-1, 1]$, where the sum of the squared length of all edges is minimized. Figure 2.21 shows this placement.
- These values define the following ordering among the nodes:

$$Z = \{a, f, d, i, g, j, b, h, e, c\}$$

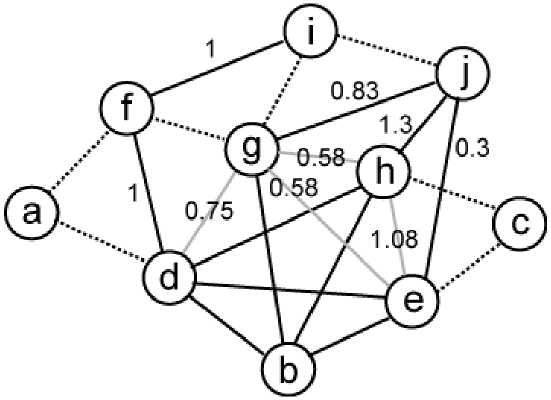


EIG Partitioning

(a) Partitioning $(\{a\}, \{f, d, i, g, j, b, h, e, c\})$:
The cut edges are (a, f) and (a, d). Thus, the cutsize is $0.5+0.5 = 1.0$. The ratio cut is $1.0/(1 \cdot 9) = 0.1111$.

(b) Partitioning $(\{a, f\}, \{d, i, g, j, b, h, e, c\})$:
The cut edges are (f, i), (f, g), (f, d) and (a, d). Thus, the cutsize is $1.0 + 0.5 + 1.0 + 0.5 = 3.0$. The ratio cut is $3.0/(2 \cdot 8) = 0.1875$.

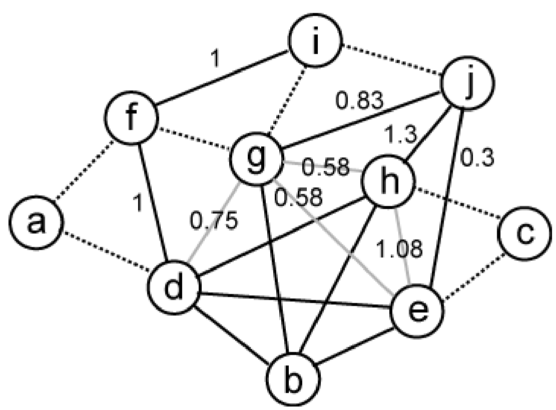
(c) Partitioning $(\{a, f, d\}, \{i, g, j, b, h, e, c\})$:
The cut edges are (f, i), (f, g), (d, g), (d, h), (d, e), and (d, b). Thus, the cutsize is $1.0 + 0.5 + 0.75 + 3 \cdot 0.25 = 3.0$. The ratio cut is $3.0/(3 \cdot 7) = 0.1429$.



EIG Partitioning (cont)

(d) Partitioning $(\{a, f, d, i\}, \{g, j, b, h, e, c\})$:
The cut edges are (i, j), (i, g), (f, g), (d, g), (d, h), (d, e), and (d, b). Thus, the cutsize is $0.5 \cdot 3 + 0.75 + 3 \cdot 0.25 = 3.0$. The ratio cut is $3.0/(4 \cdot 6) = 0.125$.

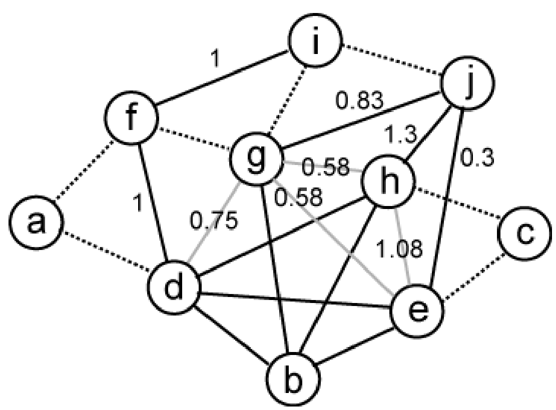
(e) Partitioning $(\{a, f, d, i, g\}, \{j, b, h, e, c\})$:
The cut edges are (i, j), (g, j), (g, h), (g, e), (g, b), (d, h), (d, e), and (d, b). Thus, the cutsize is $0.5 + 0.83 + 0.58 \cdot 2 + 0.25 \cdot 4 = 3.49$. The ratio cut is $3.49/(5 \cdot 5) = 0.1396$.



EIG Partitioning (cont)

(f) Partitioning $(\{a, f, d, i, g, j\}, \{b, h, e, c\})$:
The cut edges are (j, e), (j, h), (g, h), (g, e), (g, b), (d, h), (d, e), and (d, b). Thus, the cutsize is $0.33 + 1.33 + 0.58 \cdot 2 + 0.25 \cdot 4 = 3.82$. The ratio cut is $3.82/(6 \cdot 4) = 0.1592$.

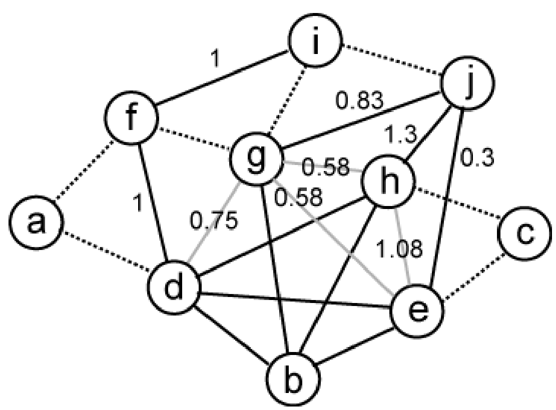
(g) Partitioning $(\{a, f, d, i, g, j, b\}, \{h, e, c\})$:
The cut edges are (j, e), (j, h), (g, h), (g, e), (d, h), (d, e), (b, h), and (b, e). Thus, the cutsize is $0.33 + 1.33 + 0.58 \cdot 2 + 0.25 \cdot 4 = 3.82$. The ratio cut is $3.82/(7 \cdot 3) = 0.1819$.



EIG Partitioning (cont)

(h) Partitioning $(\{a, f, d, i, g, j, b, h\}, \{e, c\})$:
The cut edges are (h, c), (h, e), (j, e), (g, e), (d, e), and (b, e). Thus, the cutsize is $0.5 + 1.08 + 0.33 + 0.58 + 0.25 + 0.25 = 2.99$. The ratio cut is $2.99/(8 \cdot 2) = 0.1869$.

(i) Partitioning $(\{a, f, d, i, g, j, b, h, e\}, \{c\})$:
The cut edges are (h, c) and (e, c). Thus, the cutsize is $0.5 + 0.5 = 1.0$. The ratio cut is $1.0/(9 \cdot 1) = 0.1111$.



Summary

- Good solution found:
  - {(a,f,d,g,i), (j,b,h,e,c)} is well-balanced and has low RC cost.

P_A	P_B	cutsize	ratio cut
$\{a\}$	$\{f, d, i, g, j, b, h, e, c\}$	1.0	$1.0/(1 \cdot 9) = 0.1111$
$\{a, f\}$	$\{d, i, g, j, b, h, e, c\}$	3.0	$3.0/(2 \cdot 8) = 0.1875$
$\{a, f, d\}$	$\{i, g, j, b, h, e, c\}$	3.0	$3.0/(3 \cdot 7) = 0.1429$
$\{a, f, d, i\}$	$\{g, j, b, h, e, c\}$	3.0	$3.0/(4 \cdot 6) = 0.125$
$\{a, f, d, i, g\}$	$\{j, b, h, e, c\}$	3.49	$3.49/(5 \cdot 5) = 0.1396$
$\{a, f, d, i, g, j\}$	$\{b, h, e, c\}$	3.82	$3.82/(6 \cdot 4) = 0.1592$
$\{a, f, d, i, g, j, b\}$	$\{h, e, c\}$	3.82	$3.82/(7 \cdot 3) = 0.1819$
$\{a, f, d, i, g, j, b, h\}$	$\{e, c\}$	2.99	$2.99/(8 \cdot 2) = 0.1869$
$\{a, f, d, i, g, j, b, h, e\}$	$\{c\}$	1.0	$1.0/(9 \cdot 1) = 0.1111$

Theorem

Verify that the second smallest eigenvalue is a tight lower bound of the ratio cut metric.

The eigenvalue is $\lambda = 0.6281$. It is shown in [Hagen and Kahng, 1992] that $c \geq \lambda/n$, where c is the ratio cut cost, and n is the number of nodes in the graph. Since $n = 10$ in our case, we see that $\lambda/n = 0.06281$ is smaller than all of the ratio cut values shown in Table 1.6.