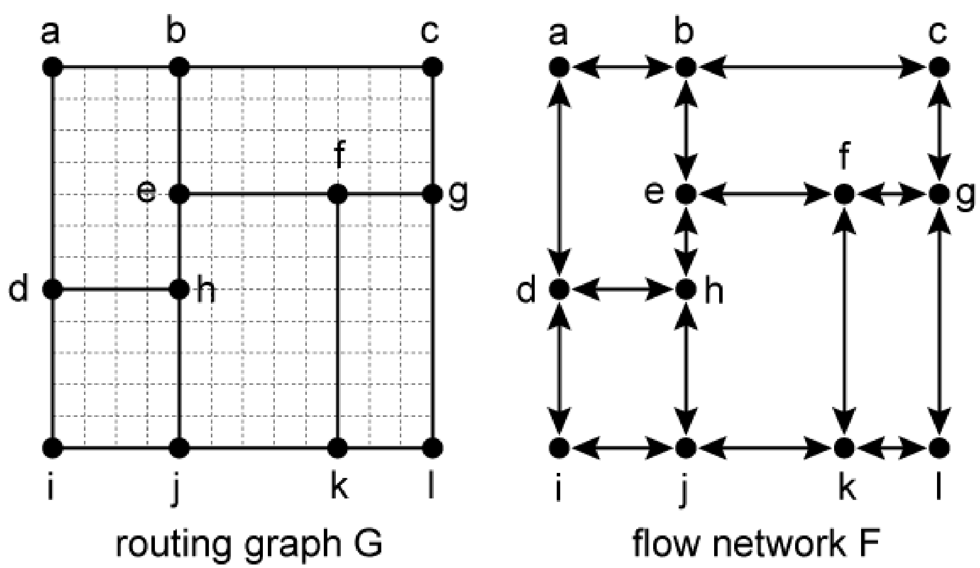
Multi-Commodity Flow Based Routing

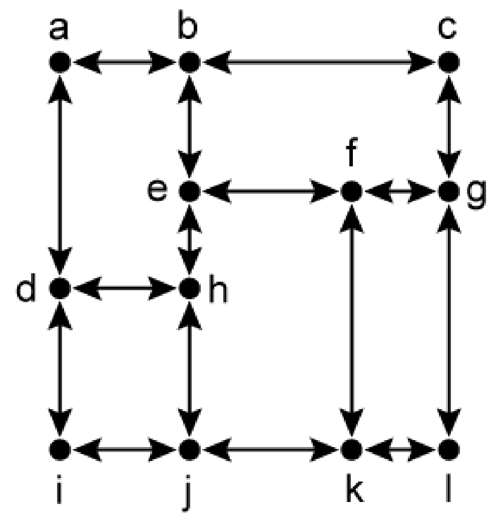
- Set up ILP formulation for MCF routing
  - Capacity of each edge in *G* is 2
  - Each edge in *G* becomes a pair of bi-directional arcs in *F*
  - $n_1 = \{a,l\}$, $n_2 = \{i,c\}$, $n_3 = \{d,f\}$, $n_4 = \{k,d\}$, $n_5 = \{g,h\}$, $n_6 = \{b,k\}$



Flow Network

- Each arc has a cost based on its length
  - Let x_e^k denote a binary variable for arc *e* w.r.t. net *k*
  - $x_e^k = 1$ means net *k* uses arc *e* in its route
  - Total number of *x*-variables: $16 \times 2 \times 6 = 192$

arc	cost	arc	cost	arc	cost	arc	cost
(a, b)	4	(b, a)	4	(b, c)	8	(c, b)	8
(d, h)	4	(h, d)	4	(e, f)	5	(f, e)	5
(f, g)	3	(g, f)	3	(i, j)	4	(j, i)	4
(j, k)	5	(k, j)	5	(k, l)	3	(l, k)	3
(a, d)	7	(d, a)	7	(d, i)	5	(i, d)	5
(b, e)	4	(e, b)	4	(e, h)	3	(h, e)	3
(h, j)	5	(j, h)	5	(f, k)	8	(k, f)	8
(c, g)	4	(g, c)	4	(g, l)	8	(l, g)	8



ILP Objective Function

- Minimize

$$4(x_{a,b}^1 + \cdots + x_{a,b}^6) + 4(x_{b,a}^1 + \cdots + x_{b,a}^6) + 8(x_{b,c}^1 + \cdots + x_{b,c}^6) +$$
$$8(x_{c,b}^1 + \cdots + x_{c,b}^6) + 4(x_{d,h}^1 + \cdots + x_{d,h}^6) + 4(x_{h,d}^1 + \cdots + x_{h,d}^6) +$$
$$5(x_{e,f}^1 + \cdots + x_{e,f}^6) + 5(x_{f,e}^1 + \cdots + x_{f,e}^6) + 3(x_{f,g}^1 + \cdots + x_{f,g}^6) +$$
$$3(x_{g,f}^1 + \cdots + x_{g,f}^6) + 4(x_{i,j}^1 + \cdots + x_{i,j}^6) + 4(x_{j,i}^1 + \cdots + x_{j,i}^6) +$$
$$5(x_{j,k}^1 + \cdots + x_{j,k}^6) + 5(x_{k,j}^1 + \cdots + x_{k,j}^6) + 3(x_{k,l}^1 + \cdots + x_{k,l}^6) +$$
$$3(x_{l,k}^1 + \cdots + x_{l,k}^6) + 7(x_{a,d}^1 + \cdots + x_{a,d}^6) + 7(x_{d,a}^1 + \cdots + x_{d,a}^6) +$$
$$5(x_{d,i}^1 + \cdots + x_{d,i}^6) + 5(x_{i,d}^1 + \cdots + x_{i,d}^6) + 4(x_{b,e}^1 + \cdots + x_{b,e}^6) +$$
$$4(x_{e,b}^1 + \cdots + x_{e,b}^6) + 3(x_{e,h}^1 + \cdots + x_{e,h}^6) + 3(x_{h,e}^1 + \cdots + x_{h,e}^6) +$$
$$5(x_{h,j}^1 + \cdots + x_{h,j}^6) + 5(x_{j,h}^1 + \cdots + x_{j,h}^6) + 8(x_{f,k}^1 + \cdots + x_{f,k}^6) +$$
$$8(x_{k,f}^1 + \cdots + x_{k,f}^6) + 4(x_{c,g}^1 + \cdots + x_{c,g}^6) + 4(x_{g,c}^1 + \cdots + x_{g,c}^6) +$$
$$8(x_{g,l}^1 + \cdots + x_{g,l}^6) + 8(x_{l,g}^1 + \cdots + x_{l,g}^6)$$

ILP Demand Constraint

- Utilize demand constant
  - $z_v^k = 1$ means node v is the source of net k ($= -1$ if sink)
  - Total number of z-constants: $12 \times 6 = 72$

From net $n_1 = \{a, l\}$, we have $z_a^1 = 1$, $z_l^1 = -1$.

From net $n_2 = \{i, c\}$, we have $z_i^2 = 1$, $z_c^2 = -1$.

From net $n_3 = \{d, f\}$, we have $z_d^3 = 1$, $z_f^3 = -1$.

From net $n_4 = \{k, d\}$, we have $z_k^4 = 1$, $z_d^4 = -1$.

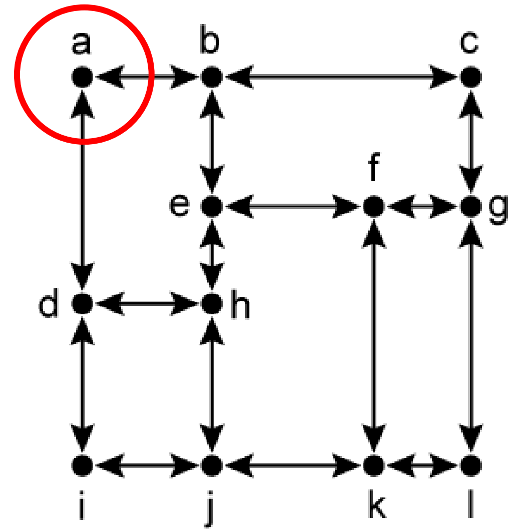
From net $n_5 = \{g, h\}$, we have $z_g^5 = 1$, $z_h^5 = -1$.

From net $n_6 = \{b, k\}$, we have $z_b^6 = 1$, $z_k^6 = -1$.

ILP Demand Constraint (cont)

- Node *a*: source of net n_1

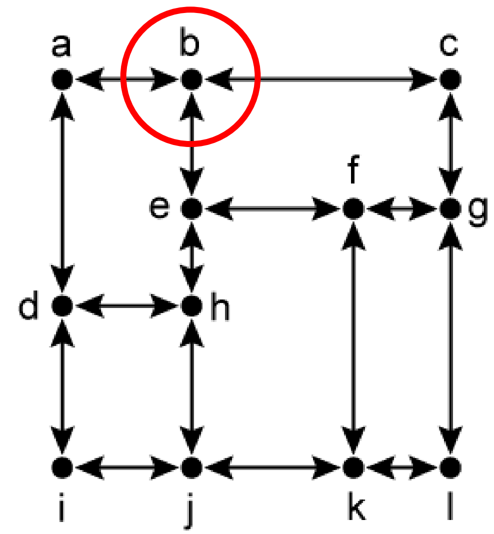
$$x^1_{a,b} + x^1_{a,d} - x^1_{b,a} - x^1_{d,a} = 1$$
$$x^2_{a,b} + x^2_{a,d} - x^2_{b,a} - x^2_{d,a} = 0$$
$$x^3_{a,b} + x^3_{a,d} - x^3_{b,a} - x^3_{d,a} = 0$$
$$x^4_{a,b} + x^4_{a,d} - x^4_{b,a} - x^4_{d,a} = 0$$
$$x^5_{a,b} + x^5_{a,d} - x^5_{b,a} - x^5_{d,a} = 0$$
$$x^6_{a,b} + x^6_{a,d} - x^6_{b,a} - x^6_{d,a} = 0$$



ILP Demand Constraint (cont)

- Node *b*: source of net n_6

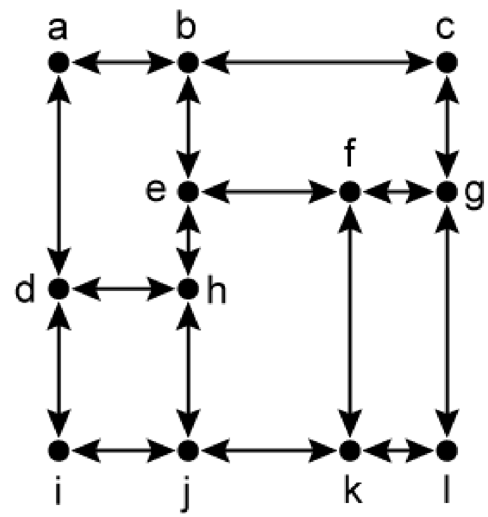
$$x_{b,a}^1 + x_{b,e}^1 + x_{b,c}^1 - x_{a,b}^1 - x_{e,b}^1 - x_{c,b}^1 = 0$$
$$x_{b,a}^2 + x_{b,e}^2 + x_{b,c}^2 - x_{a,b}^2 - x_{e,b}^2 - x_{c,b}^2 = 0$$
$$x_{b,a}^3 + x_{b,e}^3 + x_{b,c}^3 - x_{a,b}^3 - x_{e,b}^3 - x_{c,b}^3 = 0$$
$$x_{b,a}^4 + x_{b,e}^4 + x_{b,c}^4 - x_{a,b}^4 - x_{e,b}^4 - x_{c,b}^4 = 0$$
$$x_{b,a}^5 + x_{b,e}^5 + x_{b,c}^5 - x_{a,b}^5 - x_{e,b}^5 - x_{c,b}^5 = 0$$
$$x_{b,a}^6 + x_{b,e}^6 + x_{b,c}^6 - x_{a,b}^6 - x_{e,b}^6 - x_{c,b}^6 = 1$$



ILP Capacity Constraint

- Each edge in the routing graph allows 2 nets

$$x^1_{a,b} + \cdots x^6_{a,b} + x^1_{b,a} + \cdots x^6_{b,a} \leq 2$$
$$x^1_{b,c} + \cdots + x^6_{b,c} + x^1_{c,b} + \cdots + x^6_{c,b} \leq 2$$
$$x^1_{d,h} + \cdots + x^6_{d,h} + x^1_{h,d} + \cdots + x^6_{h,d} \leq 2$$
$$x^1_{e,f} + \cdots + x^6_{e,f} + x^1_{f,e} + \cdots + x^6_{f,e} \leq 2$$
$$\cdots$$
$$x^1_{h,j} + \cdots + x^6_{h,j} + x^1_{j,h} + \cdots + x^6_{j,h} \leq 2$$
$$x^1_{f,k} + \cdots + x^6_{f,k} + x^1_{k,f} + \cdots + x^6_{k,f} \leq 2$$
$$x^1_{c,g} + \cdots + x^6_{c,g} + x^1_{g,c} + \cdots + x^6_{g,c} \leq 2$$
$$x^1_{g,l} + \cdots + x^6_{g,l} + x^1_{l,g} + \cdots + x^6_{l,g} \leq 2$$

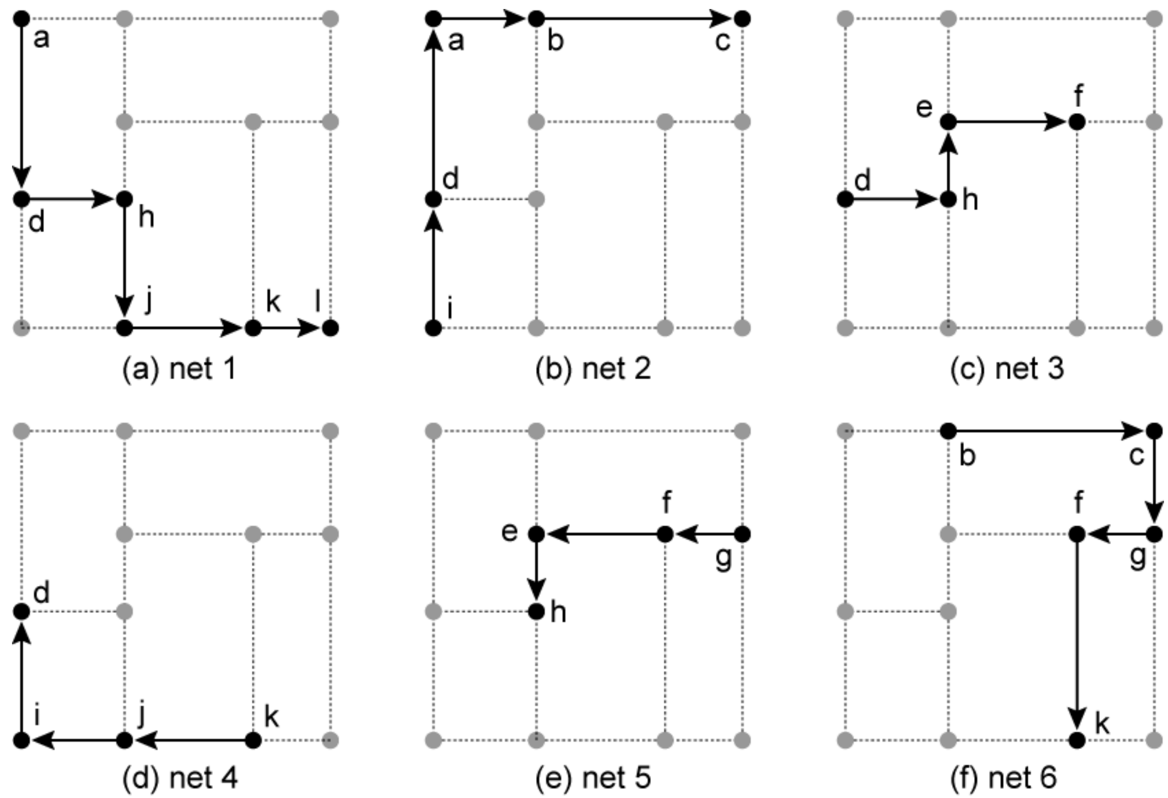


ILP Solutions

- Min-cost: 108 (= sum of WL), 22 non-zero variable
  - path for $n_1 = \{a, l\}$: we have $x^1_{a,d}$, $x^1_{d,h}$, $x^1_{h,j}$, $x^1_{j,k}$, $x^1_{k,l}$ assigned 1. Thus, the path is $a \rightarrow d \rightarrow h \rightarrow j \rightarrow k \rightarrow l$. The wirelength is 24.
  - path for $n_2 = \{i, c\}$: we have $x^2_{i,d}$, $x^2_{d,a}$, $x^2_{a,b}$, $x^2_{b,c}$ assigned 1. Thus, the path is $i \rightarrow d \rightarrow a \rightarrow b \rightarrow c$. The wirelength is 24.
  - path for $n_3 = \{d, f\}$: we have $x^3_{d,h}$, $x^3_{h,e}$, $x^3_{e,f}$ assigned 1. Thus, the path is $d \rightarrow h \rightarrow e \rightarrow f$. The wirelength is 12.
  - path for $n_4 = \{k, d\}$: we have $x^4_{k,j}$, $x^4_{j,i}$, $x^4_{i,d}$, assigned 1. Thus, the path is $k \rightarrow j \rightarrow i \rightarrow d$. The wirelength is 14.
  - path for $n_5 = \{g, h\}$: we have $x^5_{g,f}$, $x^5_{f,e}$, $x^5_{e,h}$ assigned 1. Thus, the path is $g \rightarrow f \rightarrow e \rightarrow h$. The wirelength is 11.
  - path for $n_6 = \{b, k\}$: we have $x^6_{b,c}$, $x^6_{c,g}$, $x^6_{g,f}$, $x^6_{f,k}$ assigned 1. Thus, the path is $b \rightarrow c \rightarrow g \rightarrow f \rightarrow k$. The wirelength is 23.

ILP-based MCF Routing Solution

- Net 6 is non-optimal
  - Due to congestion

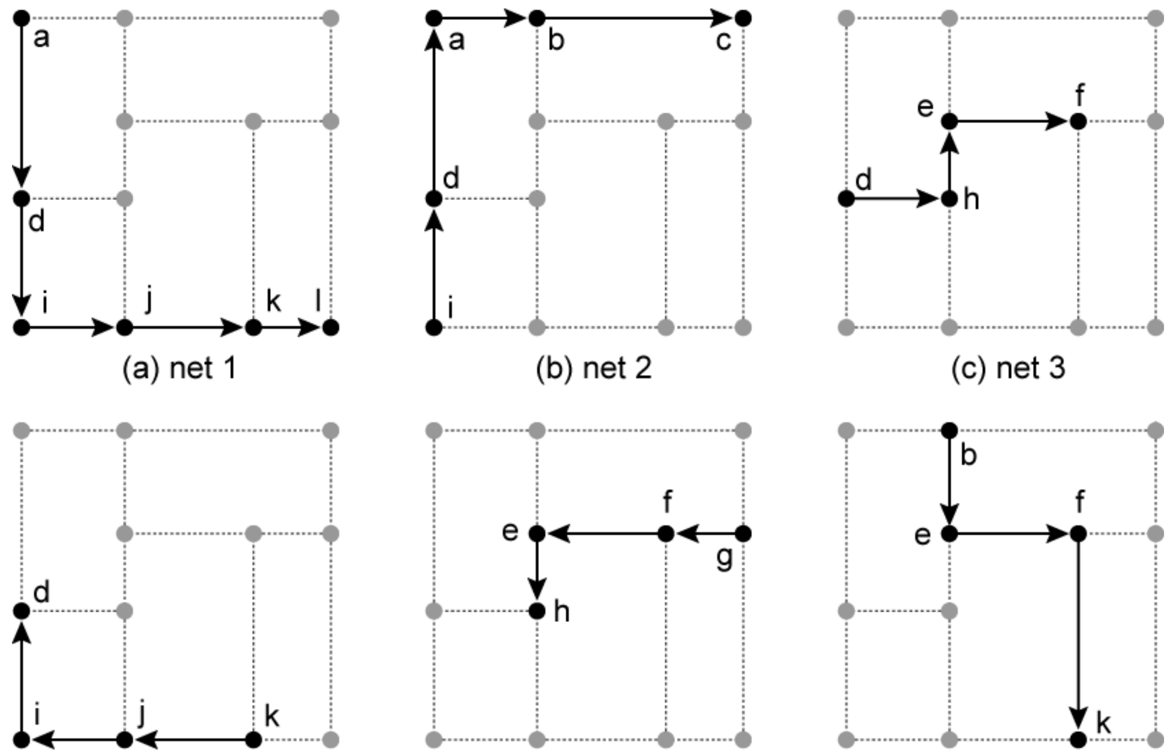


Drawback of ILP-based Method

- ILP is non-scalable
  - Runtime quickly increases with bigger problem instances
- Shragowitz and Keel presented a heuristic instead
  - Called MM (MiniMax) heuristic [1987]
  - Repeatedly perform shortest path computation and rip-up-and-reroute

MM Heuristic

- Initial set up: shortest path computation
  - Ignore capacity, some paths are not unique

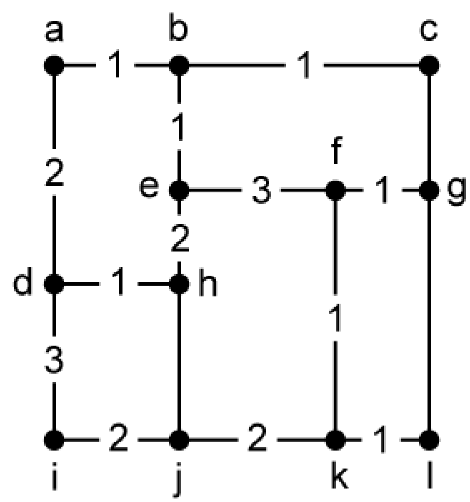


(a) net 1 (b) net 2 (c) net 3

(d) net 4 (e) net 5 (f) net 6

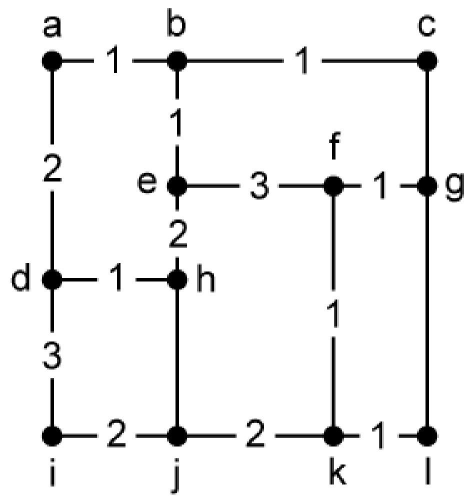
First Iteration of MM Heuristic

- Step 1
  - Capacity of channel $c(e,f)$ and $c(d,i)$ is violated
  - Max overflow $M_1 = 3 - 2 = 1 > 0$, so we proceed
  - Notation: channel $c(e,f)$ represents arc pair (e,f) and (f,e)



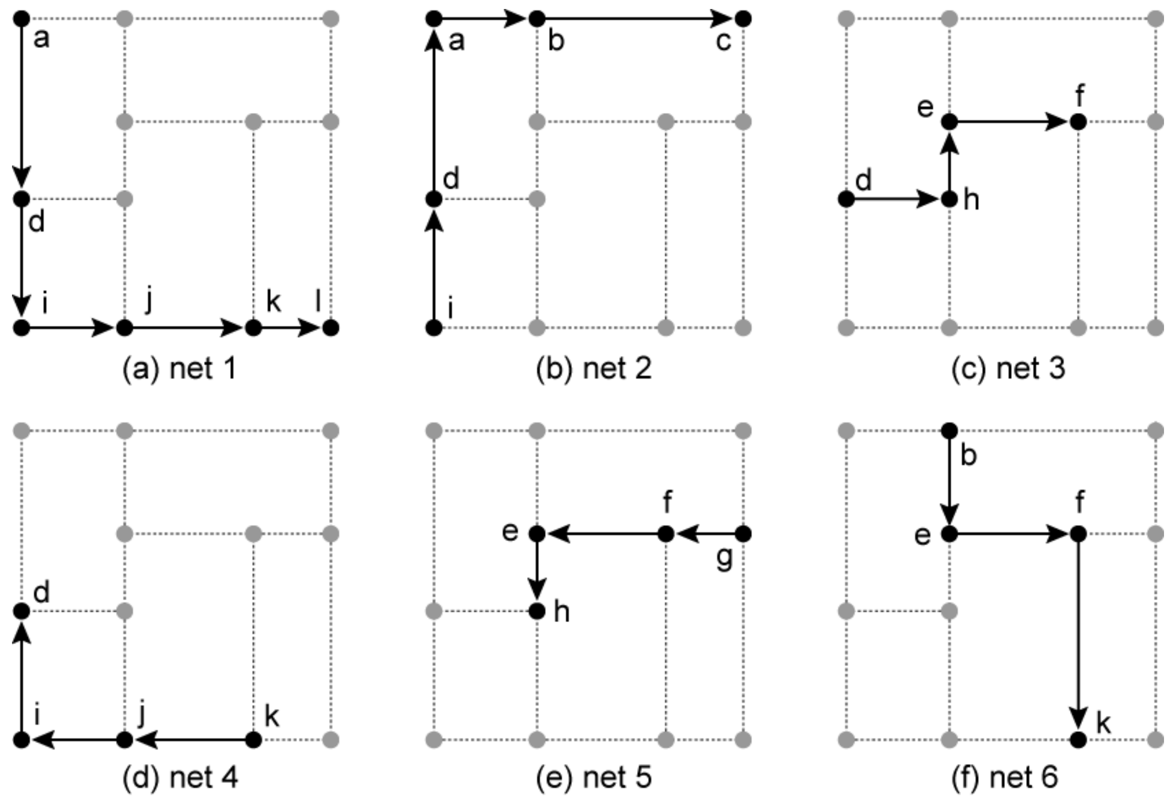
First Iteration of MM Heuristic (cont)

- Step 2
  - Set of channels with overflow of M_1: $J_1 = \{c(d,i), c(e,f)\}$
  - Set of channels with overflow of M_1 and $M_1 - 1$: $J_1^0 = \{c(a,d), c(e,h), c(i,j), c(j,k), c(d,i), c(e,f)\}$
- Step 3
  - Cost of $J_1^0 = \{c(a,d), c(e,h), c(i,j), c(j,k), c(d,i), c(e,f)\}$ is ∞



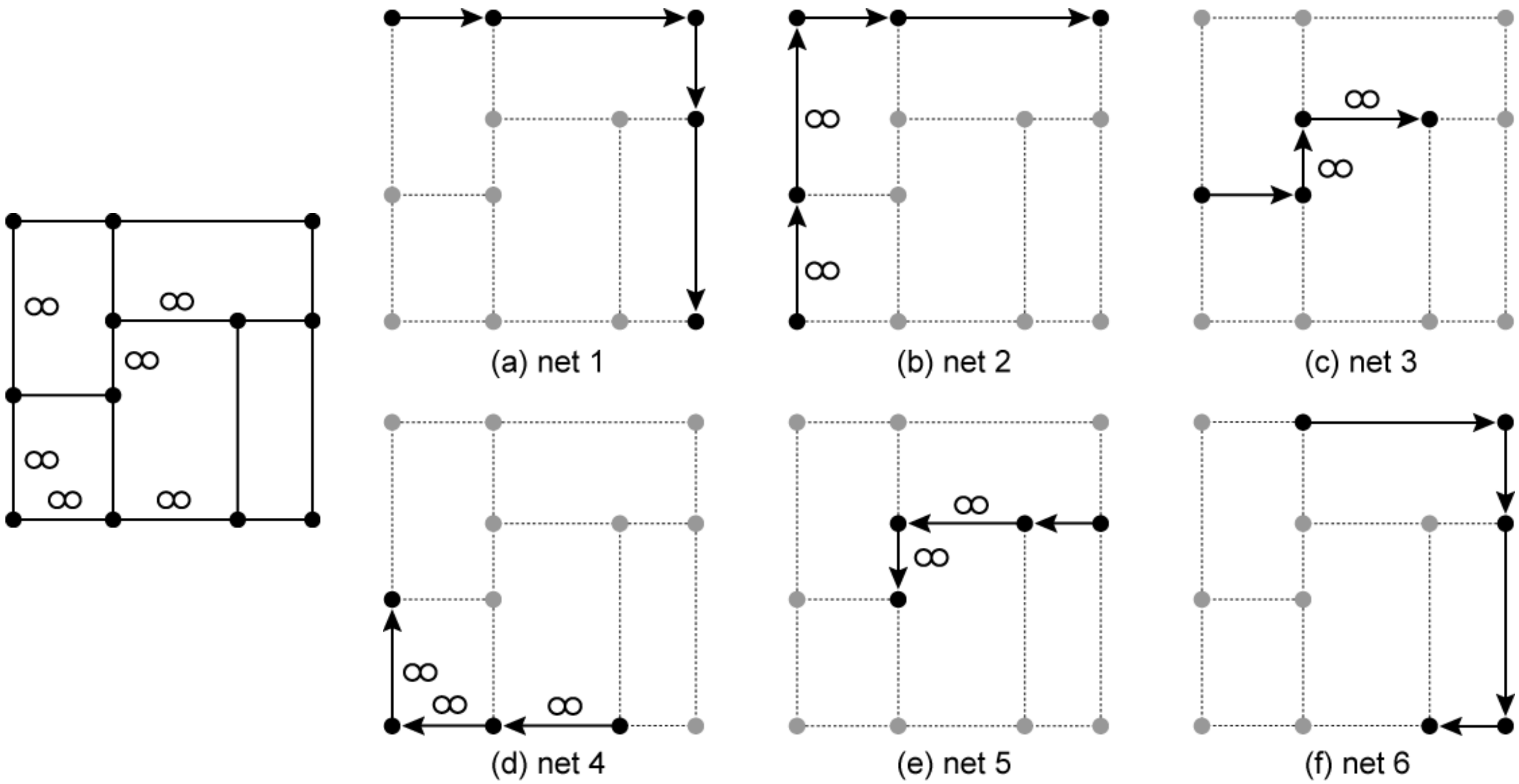
First Iteration of MM Heuristic (cont)

- Step 4
  - Set of nets using channels in J_1: $K_1 = \{n_1, n_2, n_3, n_4, n_5, n_6\}$
  - Set of nets using channels in J_1^0: $K_1^0 = K_1$



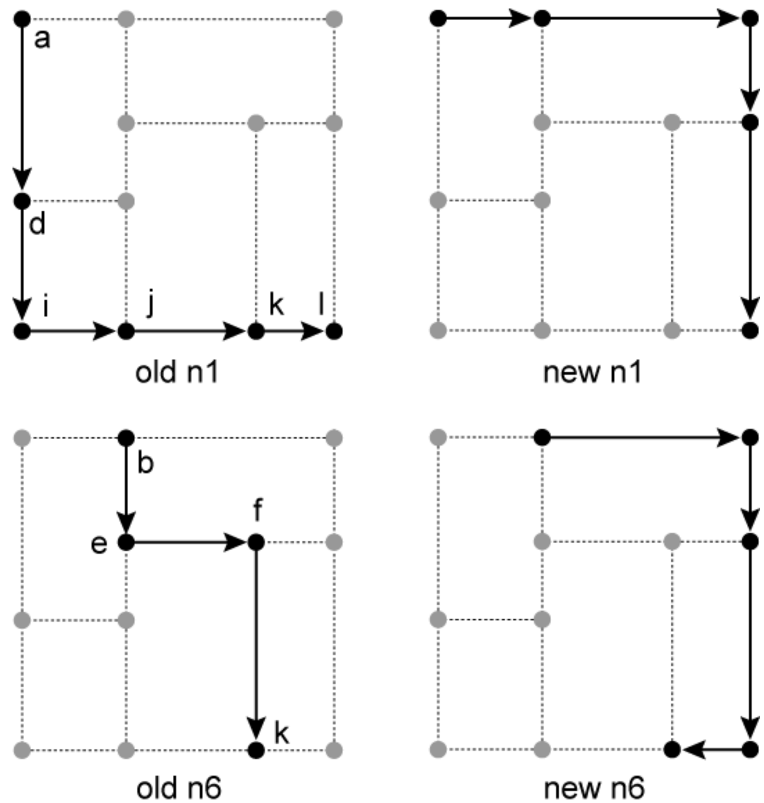
First Iteration of MM Heuristic (cont)

- Step 5
  - Compute shortest paths for nets in K_1 using new cost (= Step 3)
  - n_1 & n_6 have non-infinity cost, so we proceed



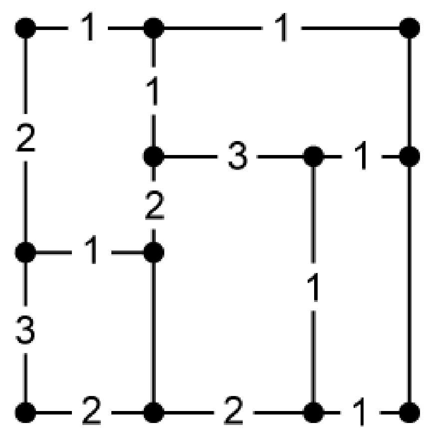
First Iteration of MM Heuristic (cont)

- Step 6
  - Net with minimum wirelength increase between n_1 & n_6: $k^0 = n_1$

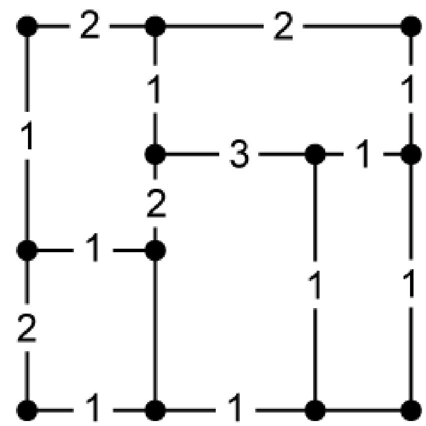


First Iteration of MM Heuristic (cont)

- Step 7
  - Use new routing for n_1
  - Wirelength didn't change, but congestion improved



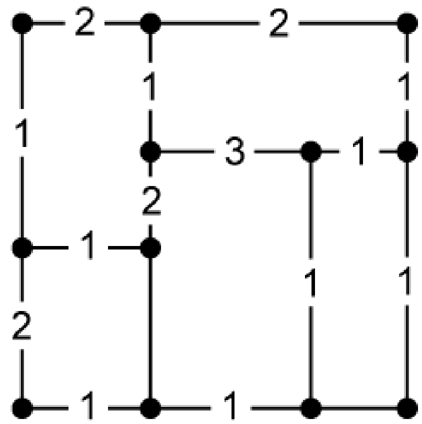
after SP computation



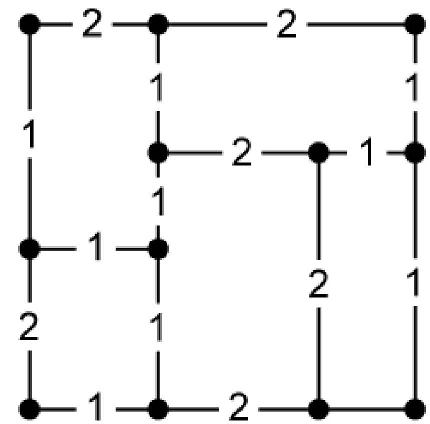
after 1st iteration of MM

Second Iteration of MM Heuristic

- Details in the book
  - Use new routing for n_3
  - Wirelength increased (due to detour in n_3), but congestion improved



after 1st iteration of MM



after 2nd iteration of MM