

# **Multi-net Routing**

**ECE6133**

**Physical Design Automation of VLSI Systems**

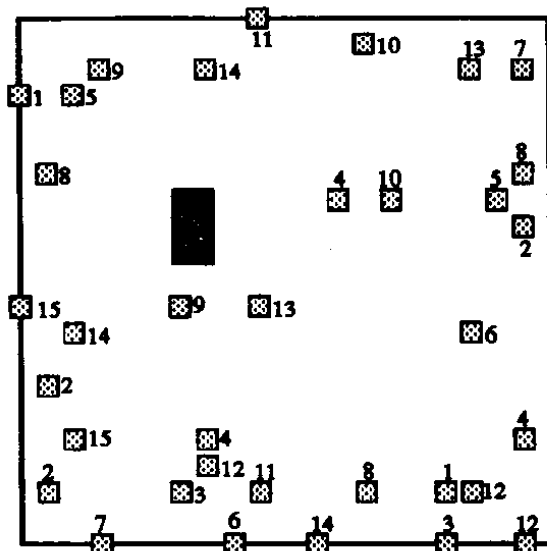
**Prof. Sung Kyu Lim**

**School of Electrical and Computer Engineering**

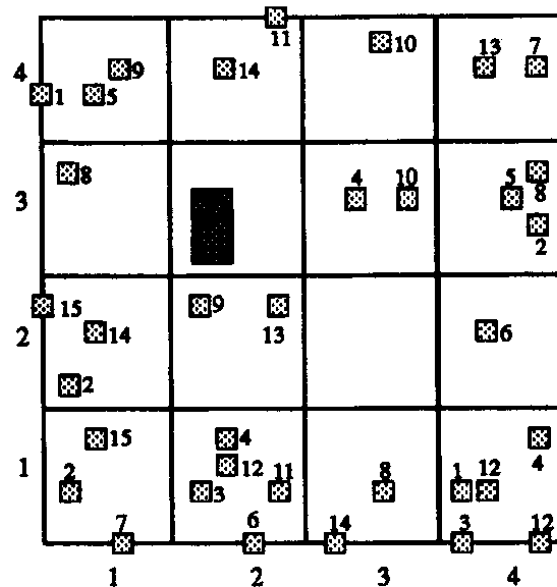
**Georgia Institute of Technology**

# Global Routing

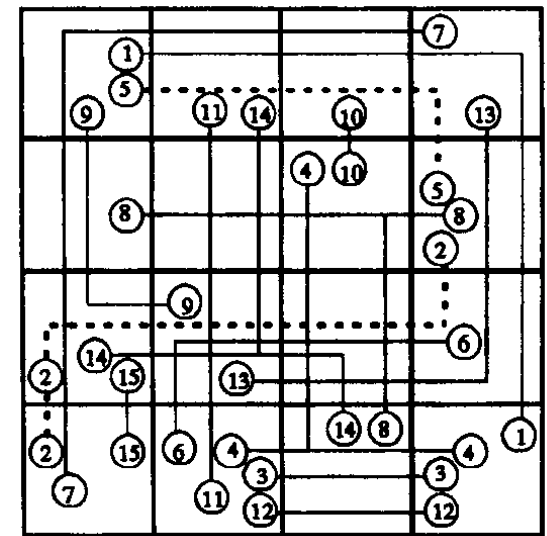
- Global routing is planning
  - Divide the routing into tiles
  - Build Steiner tree for each net
  - Routing is done in terms of tiles



Routing Problem



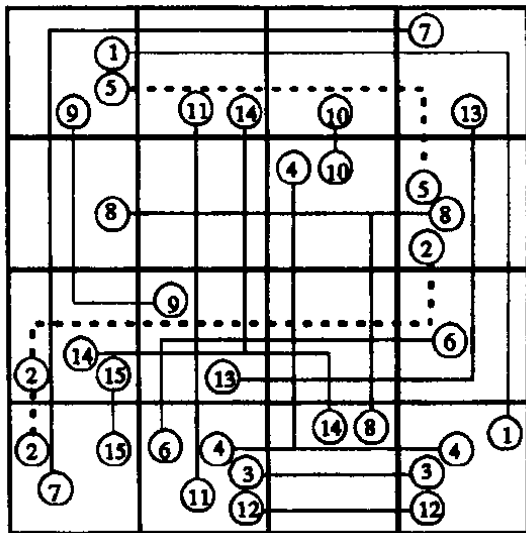
Global Routing Tiles



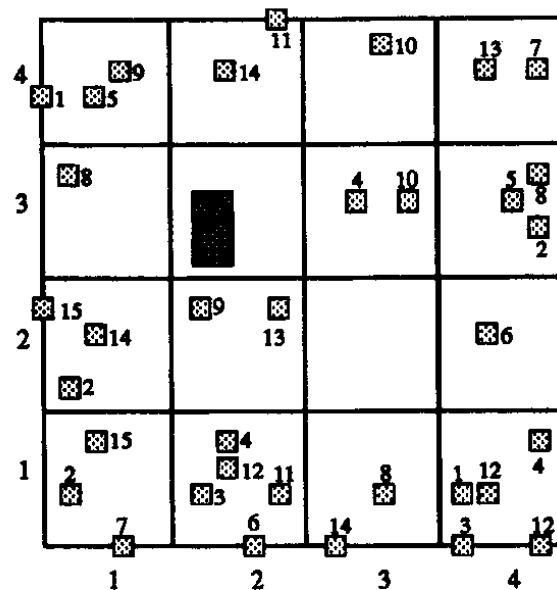
Global Routing Result

# Cross Point Assignment

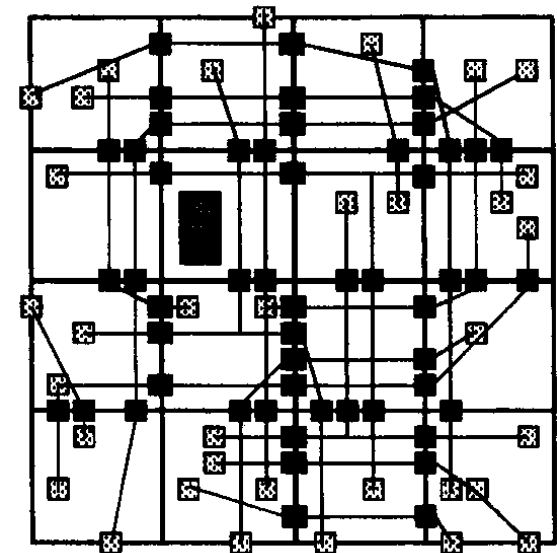
- Key step before detailed routing
  - CPA decides **pin locations along tile boundaries**
  - Key objective is routing completion, via usage, and wirelength



**Global Routing Result**  
(repeat)



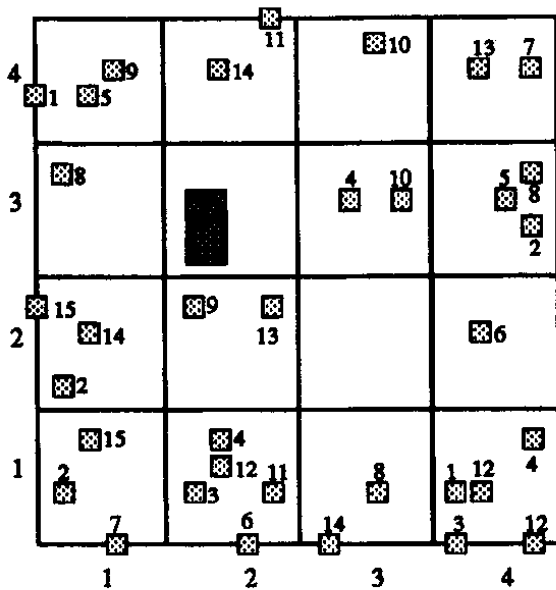
**Routing Problem**  
(repeat)



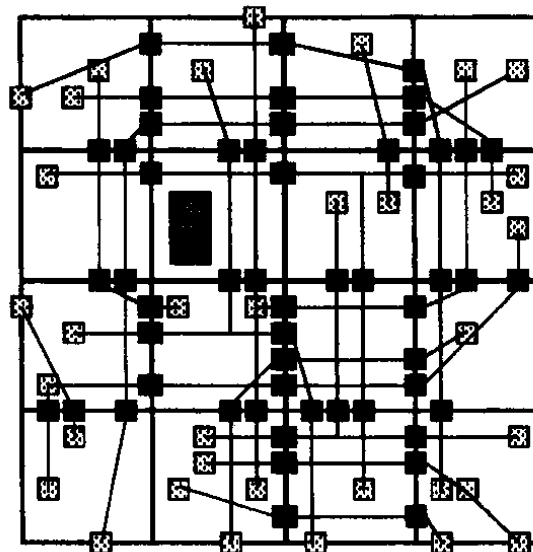
**Cross Point Assignment**

# Detailed Routing

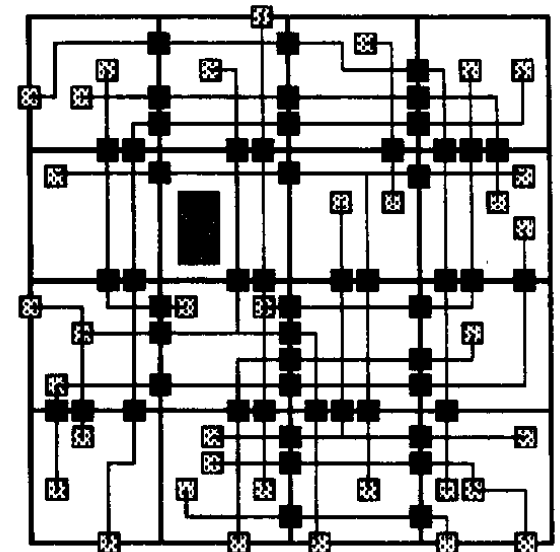
- Detailed routing decides exact topology
  - We use CPA results
  - We use the actual routing tracks and vias in each tile



**Routing Problem**  
(repeat)



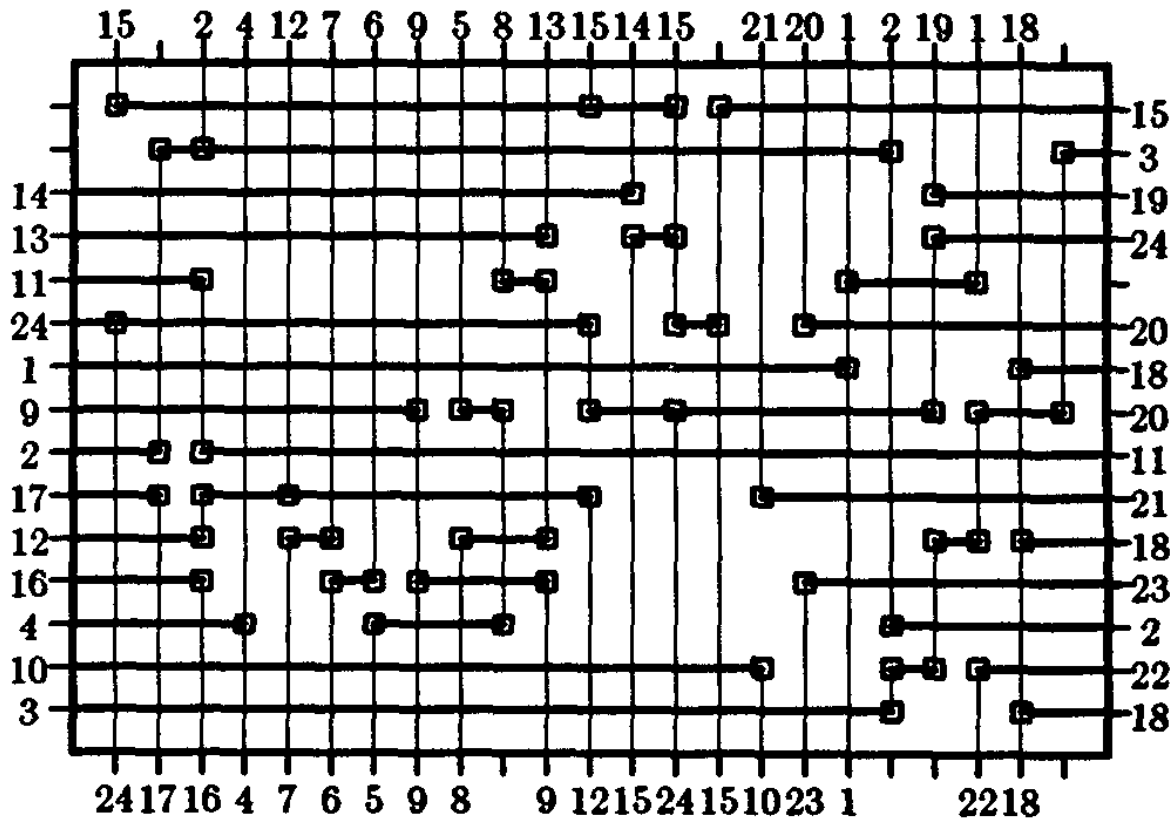
**Cross Point Assignment**



**Detailed Routing**

# Type 1: Switchbox Routing

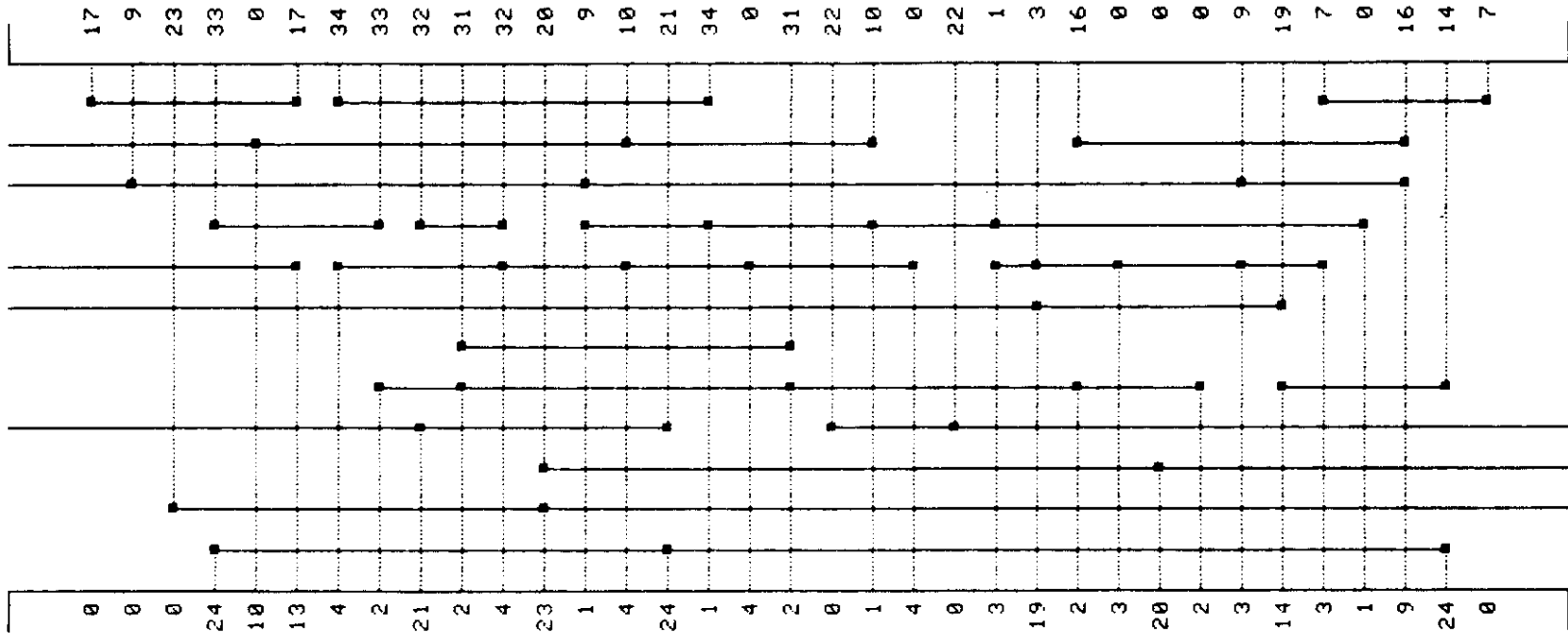
- Key problem for detailed routing
  - CPA gives **pin locations on all 4 sides**



we assume  
two metal  
layers  
(H and V)  
in this case

# Type 2: Channel Routing

- Key problem for detailed routing
  - CPA gives **pin locations on 2 sides**

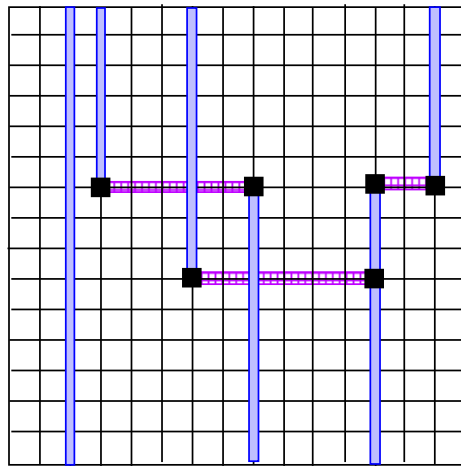


two metal layers (H and V) again

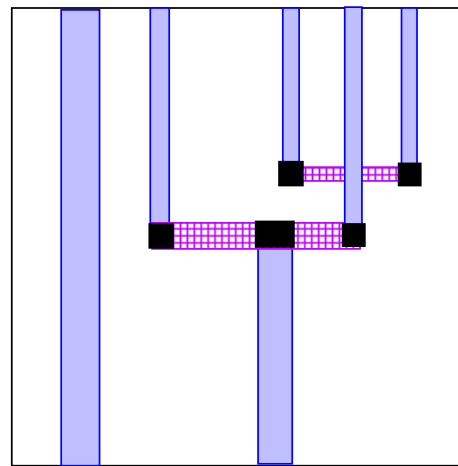
number of tracks = 12  
maximum density = 12

# Routing Models

- **Grid-based model:**
  - A grid is super-imposed on the routing region.
  - Wires follow paths along the grid lines.
- **Gridless model:**
  - Any model that does not follow this “gridded” approach.



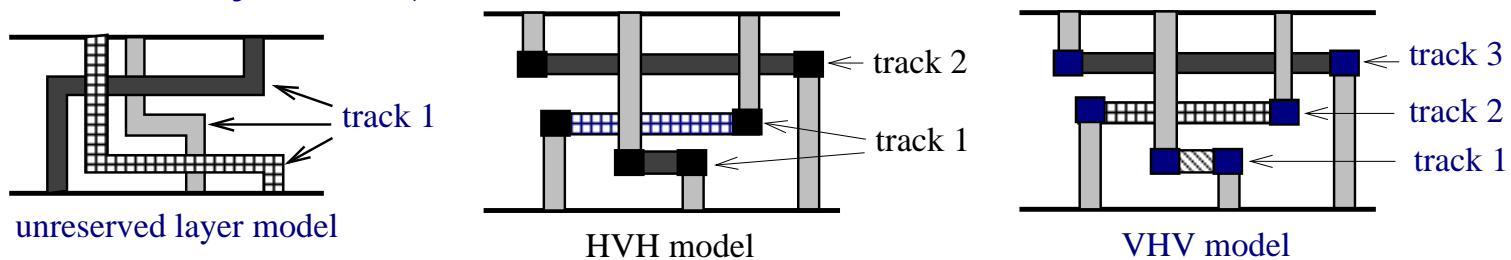
grid-based



gridless

# Models for Multi-Layer Routing

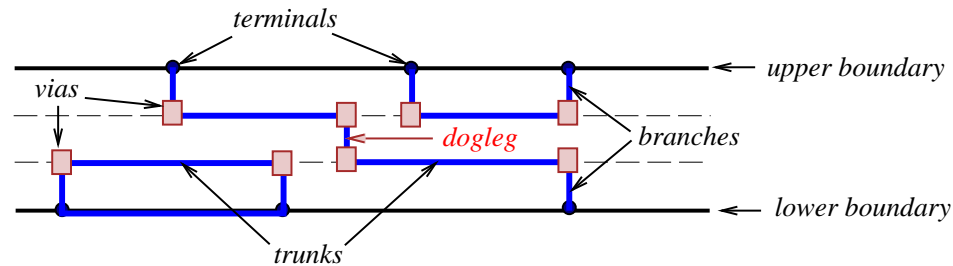
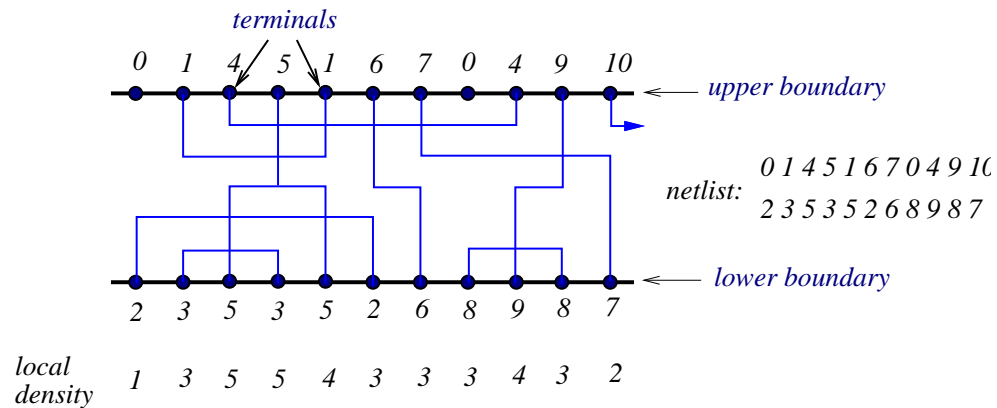
- **Unreserved layer model:** Any net segment is allowed to be placed in any layer.
- **Reserved layer model:** Certain type of segments are restricted to particular layer(s).
  - Two-layer: HV (horizontal-Vertical), VH
  - Three-layer: HVH, VHV



*3 types of 3-layer models*



# Terminology for Channel Routing Problems



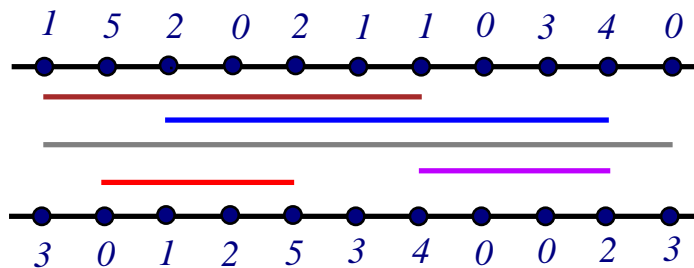
- Local density at column  $i$ : total # of nets that crosses column  $i$ .
- Channel density: maximum local density; # of horizontal tracks required  $\geq$  channel density.

# Channel Routing Problem

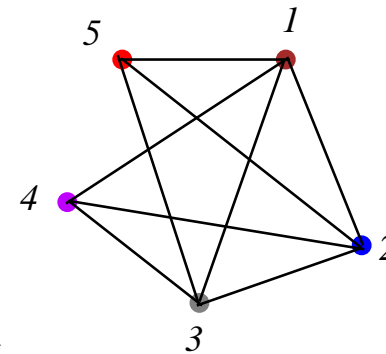
- **Assignments of horizontal segments of nets to tracks.**
- **Assignments of vertical segments to connect.**
  - horizontal segments of the same net in different tracks, and
  - the terminals of the net to horizontal segments of the net.
- **Horizontal and vertical constraints must not be violated.**
  - Horizontal constraints between two nets: The horizontal span of two nets overlaps each other.
  - Vertical constraints between two nets: There exists a column such that the terminal on top of the column belongs to one net and the terminal on bottom of the column belongs to the other net.
- **Objective: Channel height is minimized** (i.e., channel area is minimized).

# Horizontal Constraint Graph (HCG)

- HCG  $G = (V, E)$  is **undirected** graph where
  - $V = \{v_i | v_i \text{ represents a net } n_i\}$
  - $E = \{(v_i, v_j) | \text{a horizontal constraint exists between } n_i \text{ and } n_j\}$ .
- For graph  $G$ : vertices  $\Leftrightarrow$  nets; edge  $(i, j) \Leftrightarrow$  net  $i$  overlaps net  $j$ .

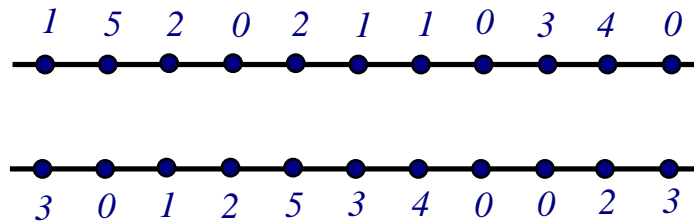


A routing problem and its HCG.

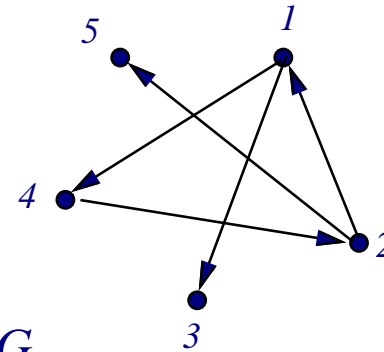


# Vertical Constraint Graph (VCG)

- VCG  $G = (V, E)$  is **directed** graph where
  - $V = \{v_i | v_i \text{ represents a net } n_i\}$
  - $E = \{(v_i, v_j) | \text{ a vertical constraint exists between } n_i \text{ and } n_j\}$ .
- For graph  $G$ : vertices  $\Leftrightarrow$  nets; edge  $i \rightarrow j \Leftrightarrow$  net  $i$  must be above net  $j$ .



*A routing problem and its VCG.*



## 2-L Channel Routing: Basic Left-Edge Algorithm

- Hashimoto & Stevens, “Wire routing by optimizing channel assignment within large apertures,” DAC-71.
- **No vertical constraint.**
- HV-layer model is used.
- **Doglegs are not allowed.**
- Treat each net as an interval.
- Intervals are sorted according to their left-end  $x$ -coordinates.
- Intervals (nets) are routed one-by-one according to the order.
- For a net, tracks are scanned from top to bottom, and the first track that can accommodate the net is assigned to the net.
- Optimality: produces a routing solution with the minimum # of tracks (if no vertical constraint).

## Basic Left-Edge Algorithm

**Algorithm: Basic\_Left-Edge( $U, track[j]$ )**

$U$ : set of unassigned intervals (nets)  $I_1, \dots, I_n$ ;

$I_j = [s_j, e_j]$ : interval  $j$  with left-end  $x$ -coordinate  $s_j$  and right-end  $e_j$ ;

$track[j]$ : track to which net  $j$  is assigned.

1 **begin**

2  $U \leftarrow \{I_1, I_2, \dots, I_n\}$ ;

3  $t \leftarrow 0$ ;

4 **while** ( $U \neq \emptyset$ ) **do**

5  $t \leftarrow t + 1$ ;

6  $watermark \leftarrow 0$ ;

7 **while** (there is an  $I_j \in U$  s.t.  $s_j > watermark$ ) **do**

8     Pick the interval  $I_j \in U$  with  $s_j > watermark$ ,  
   nearest  $watermark$ ;

9      $track[j] \leftarrow t$ ;

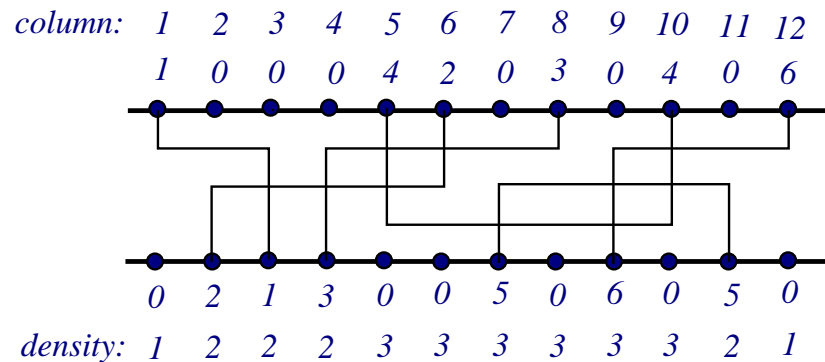
10     $watermark \leftarrow e_j$ ;

11     $U \leftarrow U - \{I_j\}$ ;

12 **end**

## Basic Left-Edge Example

- $U = \{I_1, I_2, \dots, I_6\}$ ;  $I_1 = [1, 3]$ ,  $I_2 = [2, 6]$ ,  $I_3 = [4, 8]$ ,  $I_4 = [5, 10]$ ,  $I_5 = [7, 11]$ ,  $I_6 = [9, 12]$ .
- $t = 1$ :
  - Route  $I_1$ : *watermark* = 3;
  - Route  $I_3$ : *watermark* = 8;
  - Route  $I_6$ : *watermark* = 12;
- $t = 2$ :
  - Route  $I_2$ : *watermark* = 6;
  - Route  $I_5$ : *watermark* = 11;
- $t = 3$ : Route  $I_4$



# Constrained Left-Edge Algorithm

**Algorithm: Constrained\_Left-Edge( $U, track[j]$ )**

$U$ : set of unassigned intervals (nets)  $I_1, \dots, I_n$ ;

$I_j = [s_j, e_j]$ : interval  $j$  with left-end  $x$ -coordinate  $s_j$  and right-end  $e_j$ ;

$track[j]$ : track to which net  $j$  is assigned.

1 **begin**

2  $U \leftarrow \{I_1, I_2, \dots, I_n\}$ ;

3  $t \leftarrow 0$ ;

4 **while** ( $U \neq \emptyset$ ) **do**

5  $t \leftarrow t + 1$ ;

6  $watermark \leftarrow 0$ ;

7 **while** (there is an **unconstrained**  $I_j \in U$  s.t.  $s_j > watermark$ ) **do**

8     Pick the interval  $I_j \in U$  that is unconstrained,  
   with  $s_j > watermark$ , nearest  $watermark$ ;

9      $track[j] \leftarrow t$ ;

10     $watermark \leftarrow e_j$ ;

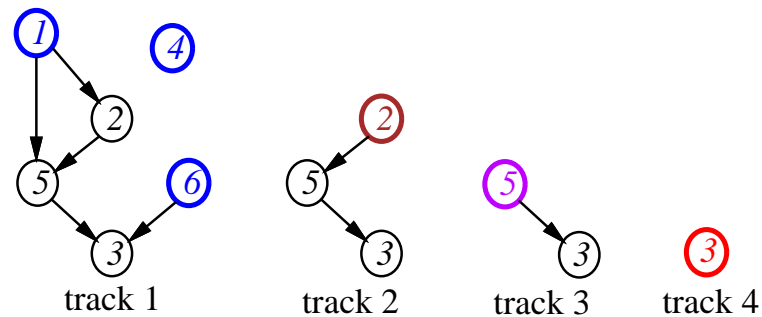
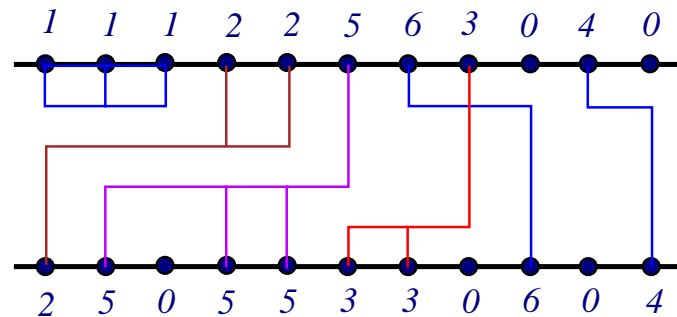
11     $U \leftarrow U - \{I_j\}$ ;

12 **end**



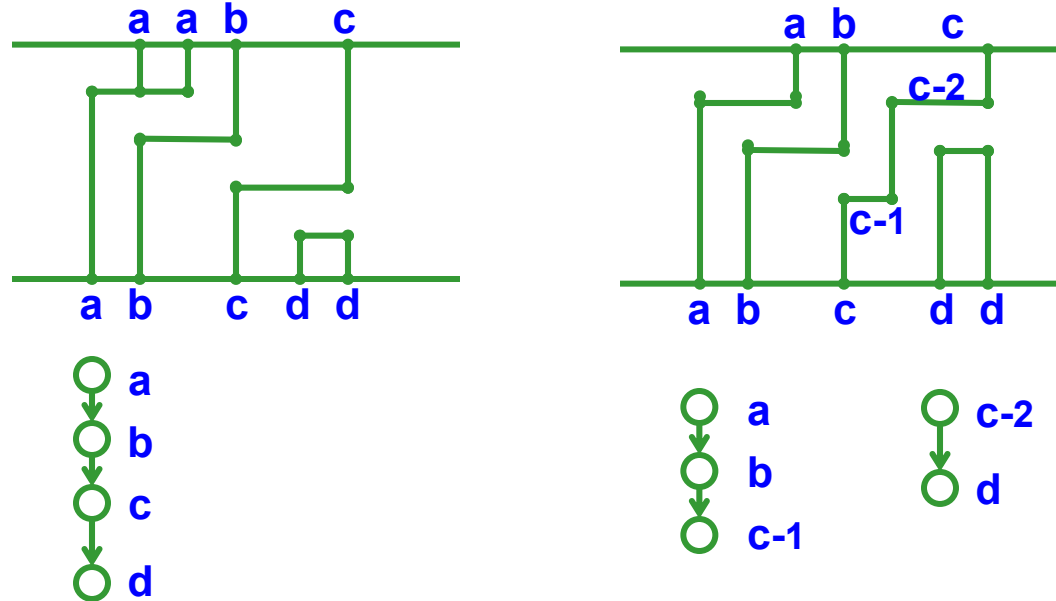
## Constrained Left-Edge Example

- $I_1 = [1, 3]$ ,  $I_2 = [1, 5]$ ,  $I_3 = [6, 8]$ ,  $I_4 = [10, 11]$ ,  $I_5 = [2, 6]$ ,  $I_6 = [7, 9]$ .
- Track 1: Route  $I_1$  (cannot route  $I_3$ ); Route  $I_6$ ; Route  $I_4$ .
- Track 2: Route  $I_2$ ; cannot route  $I_3$ .
- Track 3: Route  $I_5$ .
- Track 4: Route  $I_3$ .

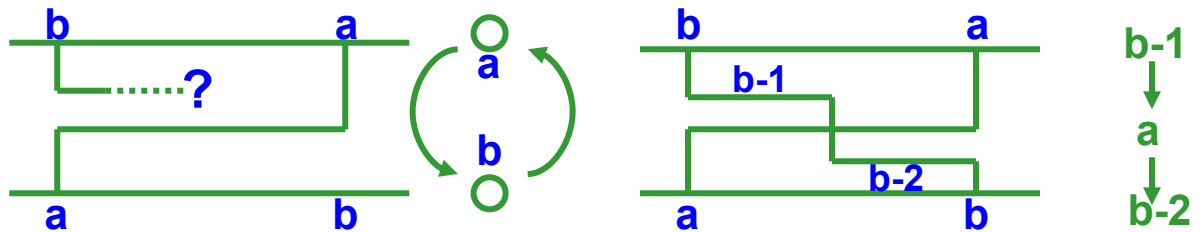


# Doglegs in Channel Routing

☞ Doglegs may reduce the longest path in VCG

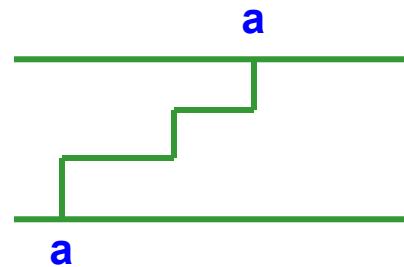
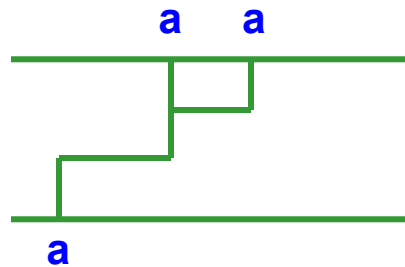


☞ Doglegs break cycles in VCG



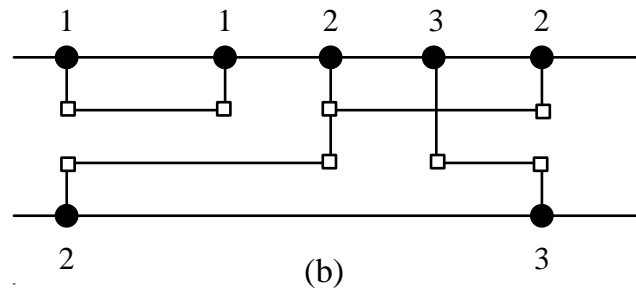
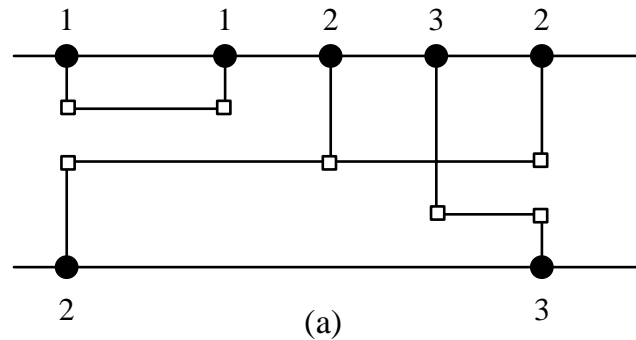
# Doglegs in Channel Routing (Cont'd)

## 🌀 Restricted Dogleg vs unrestricted dogleg



## Dogleg Router

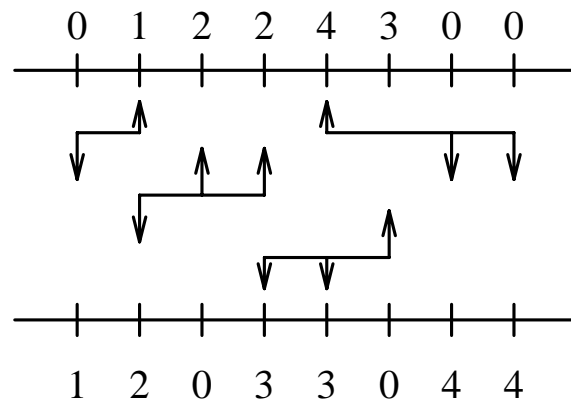
- Drawback of LEA: the entire net is on a single track.
- Doglegs are used to place parts of a net on different tracks, thereby minimizing channel height.



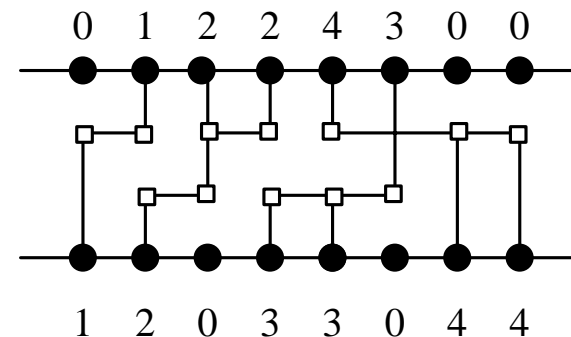
Using a dogleg to reduce channel height

## Dogleg Router

- Each Multi-terminal net is broken into a set of two-terminal nets.
- Two parameters are used to control routing:
  1. range: Determine the number of consecutive two-terminal subnets of the same net that can be placed on the same track.
  2. routing sequence: Specifies the starting position and the direction of routing along the channel.
- Modified LEA is applied to each subnet.



(a)

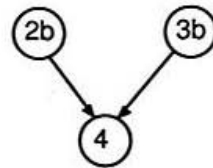
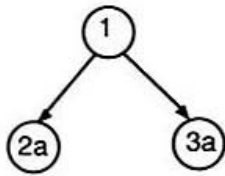
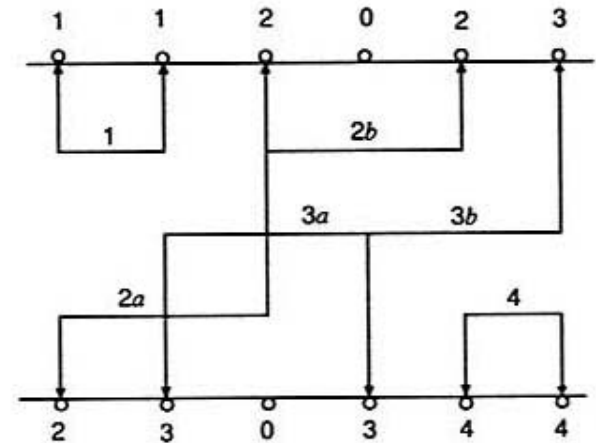
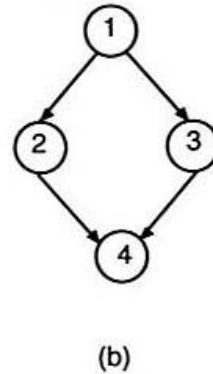
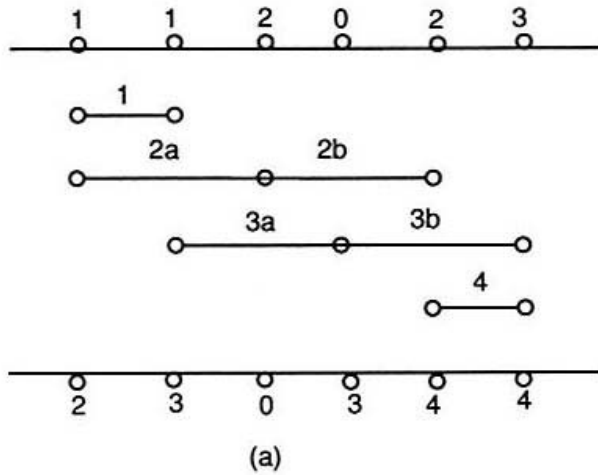


(b)

### Example of Dogleg Router

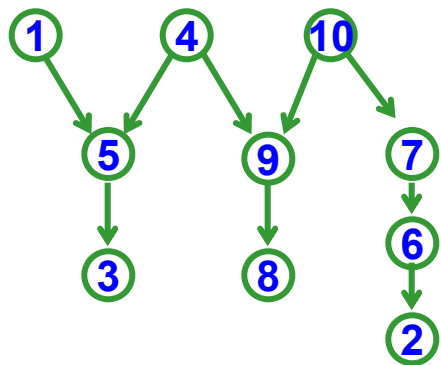
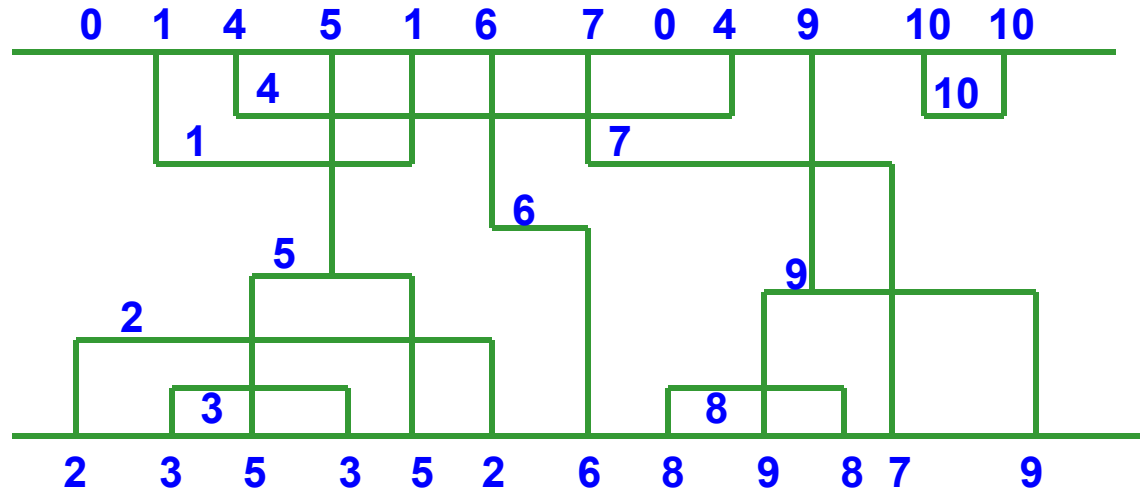
# Dogleg Router: Example

- Decompose multi-terminal nets into two-terminal nets

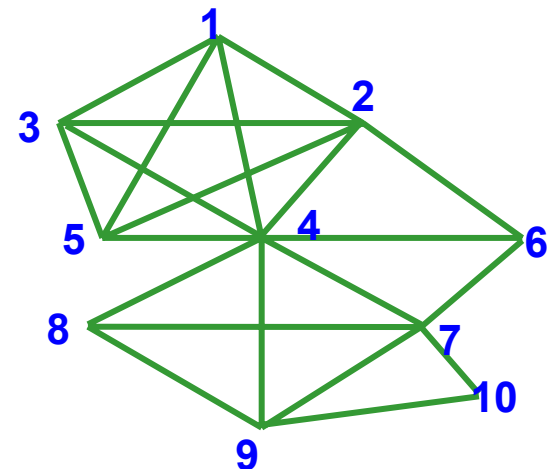


(c)

# Characterizing Channel Routing Problem



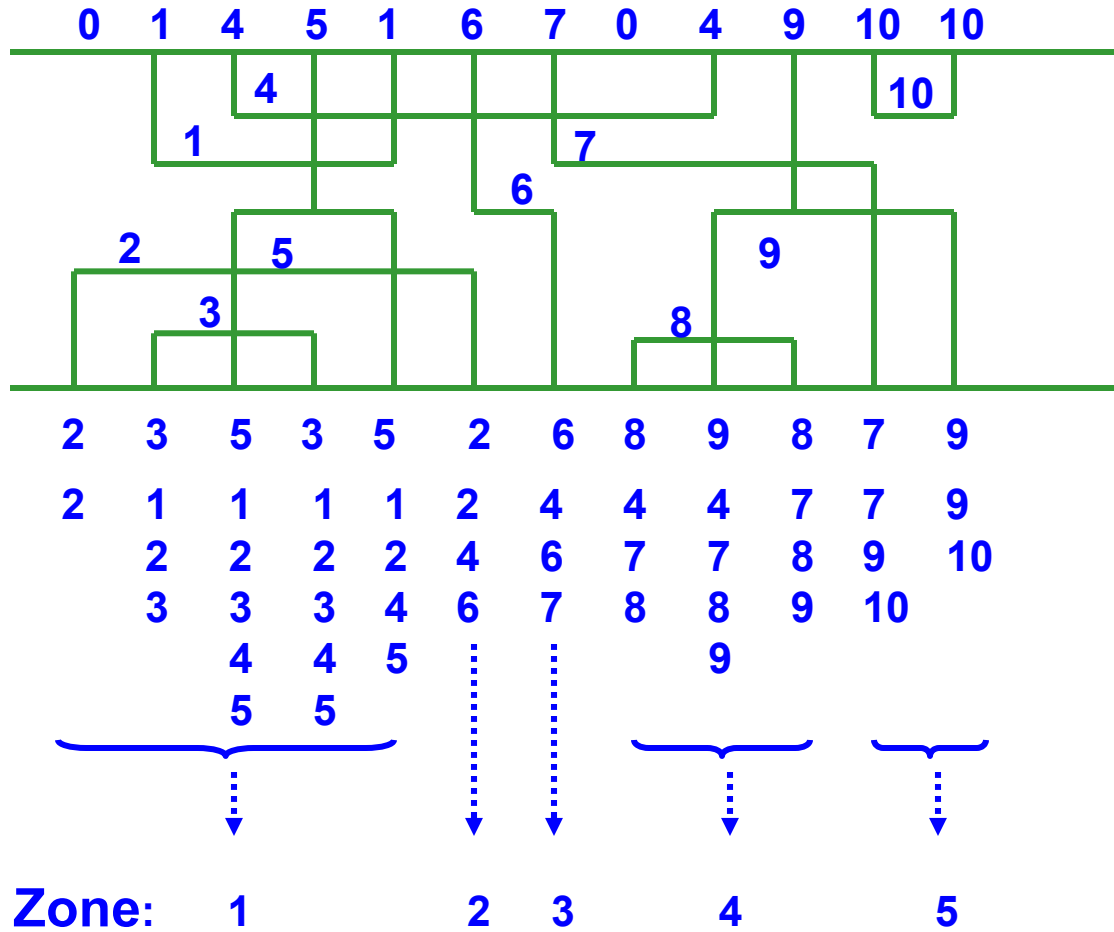
Vertical constraint graph  $G_v$



Horizontal constraint graph

The channel routing problem is completely characterized by the vertical constraint graph and the horizontal constraint graph.

# Zone Representation of Horizontal Segments





# Zone Representation of Horizontal Segments(Cont'd)

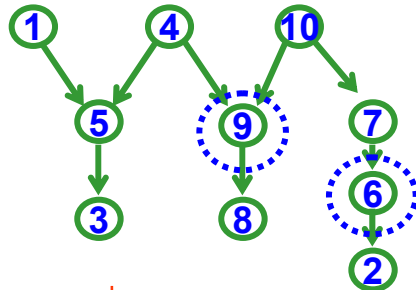
## Zone representation

**S(i): set of nets intersect column i**

☞ we only need to consider those s(i)s which are maximal

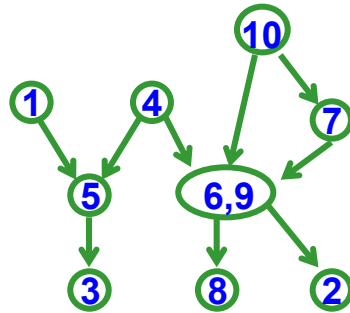
☞ **Zone**  $\leftrightarrow$  maximal clique in the horizontal constraint graph

# Merging of Nets



1		7		
2			8	
3			9	
4				
5	6			10

◇ and ⊕ can be merged



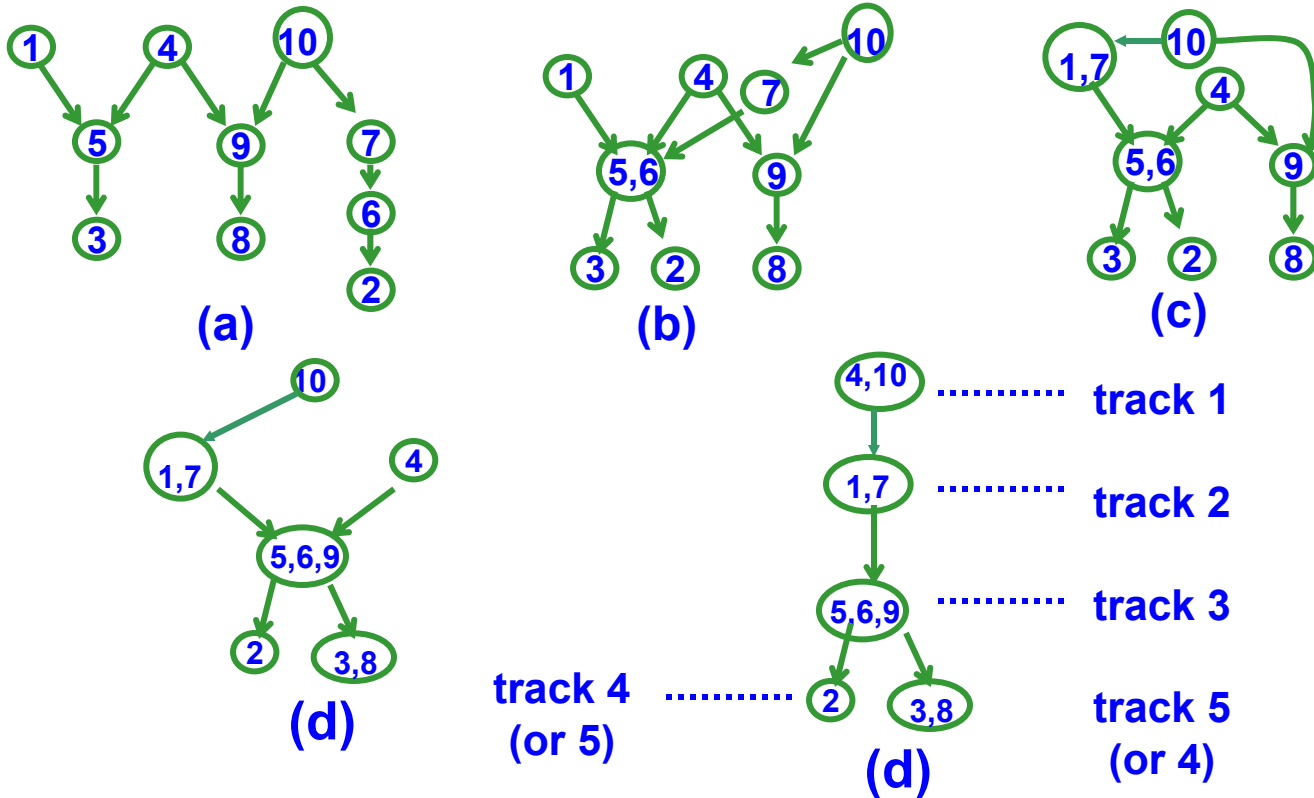
1		7		
2			8	
3				
4				10
5		6,9		

Updated graph and zone rep

Net i and net j can be merged if

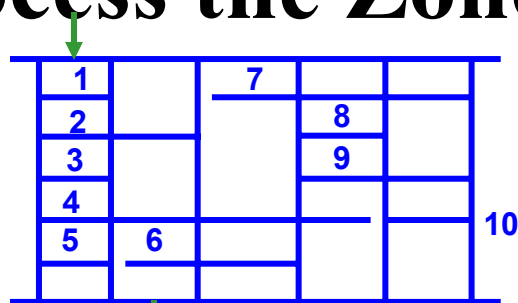
- (a) there is no path (directed connecting them in VCG;
- (b) the two nets do not overlap

# Change of VCGs



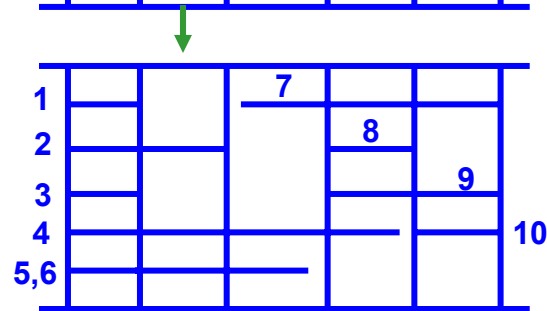
How to choose two feasible nets to merge?  
 ⇒ Determine the quality of the solutions

# Process the Zones Sequentially



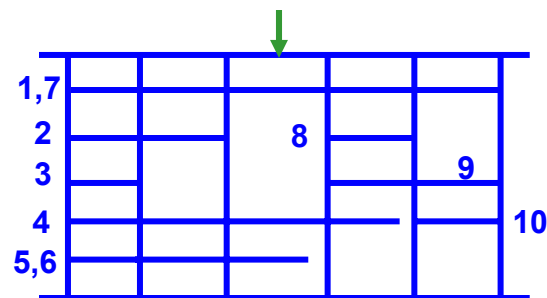
LEFT={1,3,5}

RIGHT={6}



LEFT={1,2,3}

RIGHT={7}

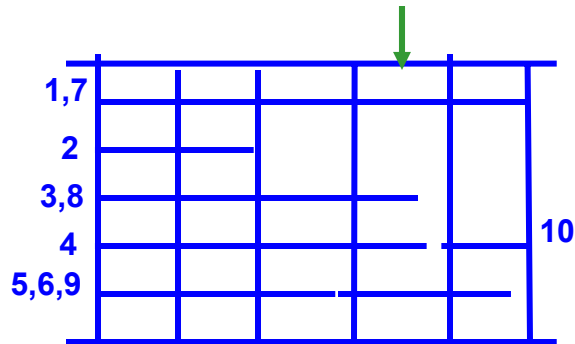


LEFT={2,3,5,6}

RIGHT={8,9}

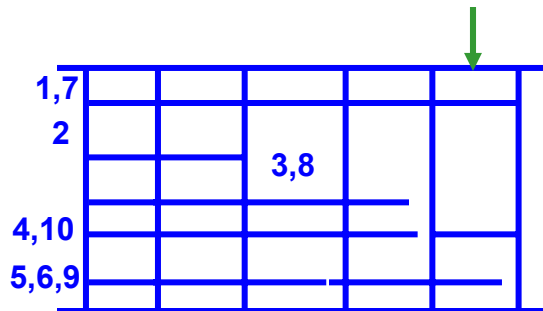
# Process the Zones Sequentially

(Cont'd)



LEFT={2,3.8,4}

RIGHT={10}

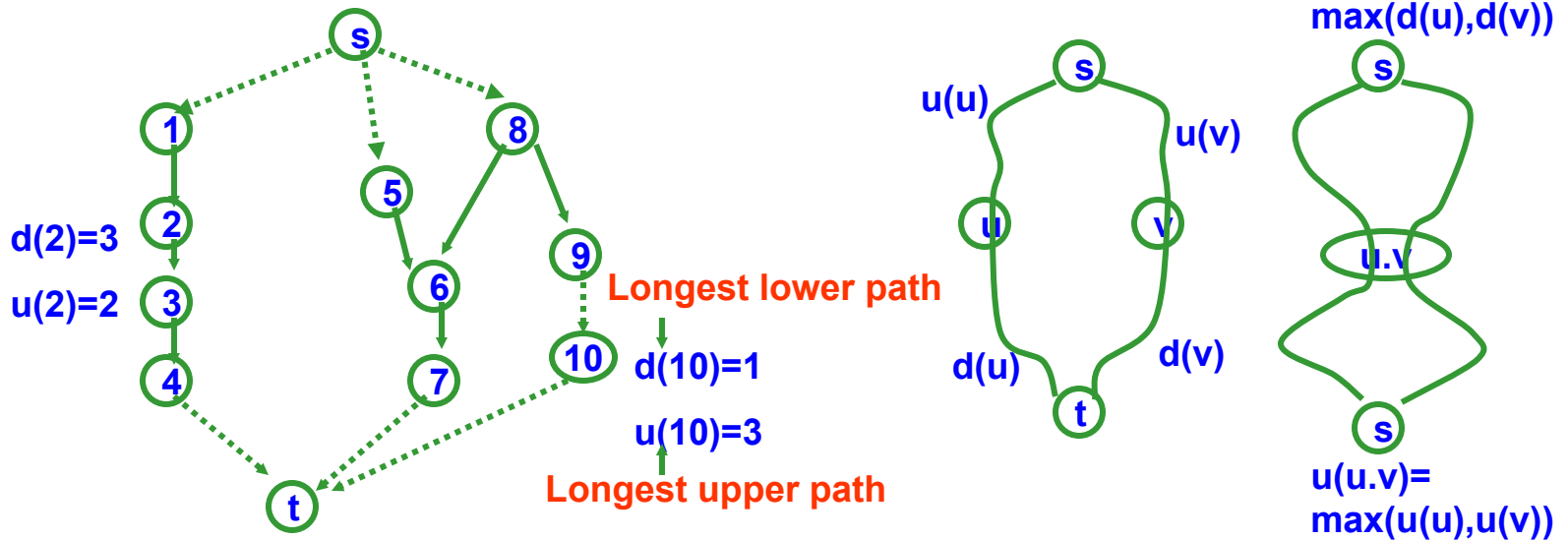


LEFT={1.7, 2, 3.8, 4.10, 5.6.9}

RIGHT=  $\phi$

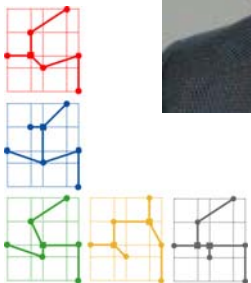
# First Approach

- ☞ Merge LEFT and RIGHT so as to minimize the increase of the longest path length in the VCG
- ☞ Heuristic rule to select nets to merge sequentially



# What to Choose from P/Q?

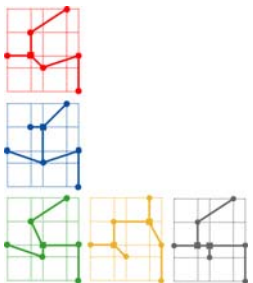
The purpose here is to minimize the length of the longest path after merger. However, it will be too time consuming to find an exact minimum merger, hence a heuristic merging algorithm will be given. Let us introduce some basic intuitive ideas. First, a node  $m \in Q$  is chosen, which lies on the longest path before merger; furthermore, it is farthest away from either  $s$  or  $t$ . Next, a node  $n \in P$  is chosen such that the increase of the longest path after merger is minimum. If there are two or more nodes which will result in a minimum increase we choose  $n$  such that  $u(n) + d(n)$  is maximum or nearly maximum and that the condition  $u(m)/d(m) = u(n)/d(n)$  is satisfied or nearly satisfied. These can be implemented by introducing the following:



# Heuristic

## *Merging Algorithm*

```
given  $P, Q$ ;  
  begin;  
a1:   while  $Q$  is not empty do;  
      begin;  
a2:   among  $Q$ , find  $m^*$  which maximizes  $f(m)$ ;  
a3:   among  $P$ , find  $n^*$  which minimizes  $g(n, m^*)$ ,  
      and which is neither ancestor nor descendent  
      of  $m^*$ ;  
a4:   merge  $n^*$  and  $m^*$ ;  
a5:   remove  $n^*$  and  $m^*$  from  $P$  and  $Q$ ,  
      respectively;  
      end;  
  end;  
end;
```





# Formulas

(1) for  $m \in Q$

$$f(m) = C_\infty * \{u(m) + d(m)\} + \max \{u(m), d(m)\},$$

$$C_\infty \gg 1$$

lies on the longest path before merge,  
farthest away from  $s$  or  $t$

(2) for  $n \in P, m \in Q$

$$g(n, m) = C_\infty * h(n, m)$$

$$- \{ \sqrt{u(m) * u(n)} + \sqrt{d(m) * d(n)} \}$$

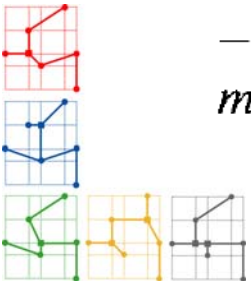
where

increase of longest path after merge is minimum,  
 $u(n)+d(n)$  maximized and  $u(m)/d(m) = u(n)/d(n)$

$$h(n, m) = \max \{u(n), u(m)\} + \max \{d(n), d(m)\}$$

$$- \max \{u(n) + d(n), u(m) + d(m)\}$$

–the increase of the longest path length passing through  $n$  or  $m$ , by merging of  $n$  and  $m$ .



# Results

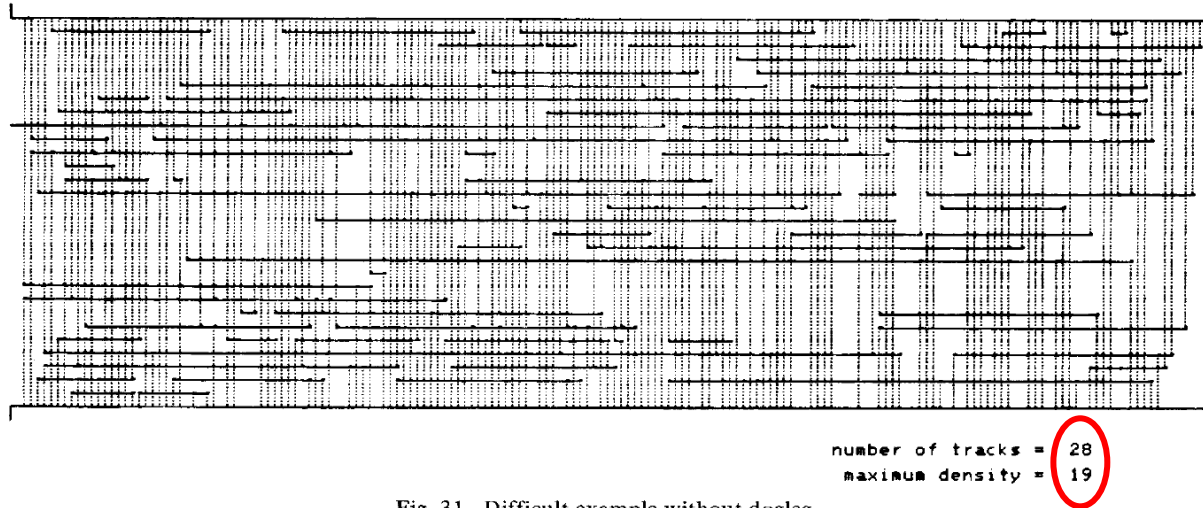


Fig. 31. Difficult example without dogleg.

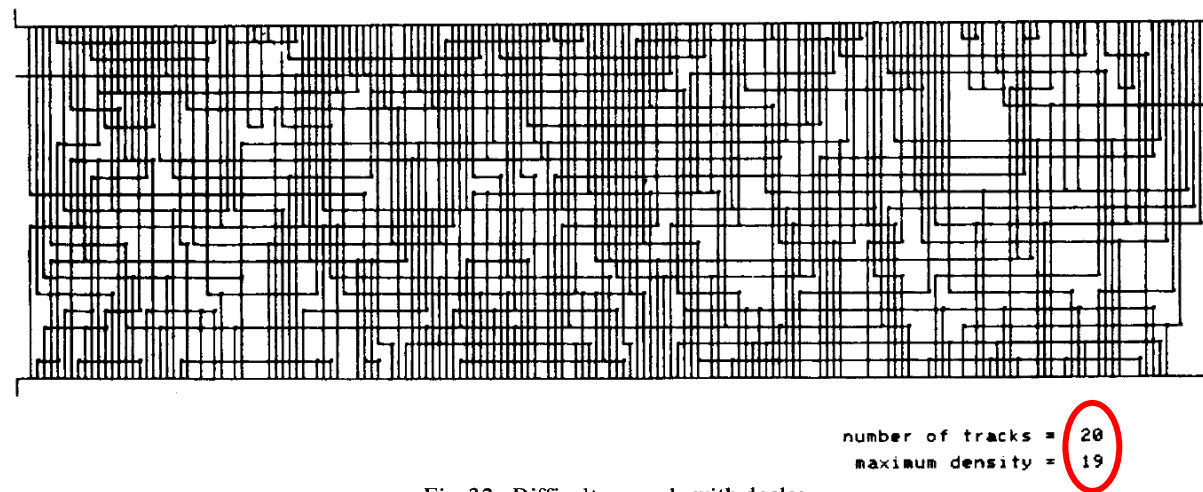
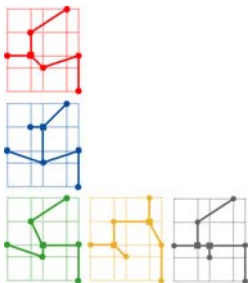


Fig. 32. Difficult example with dogleg.

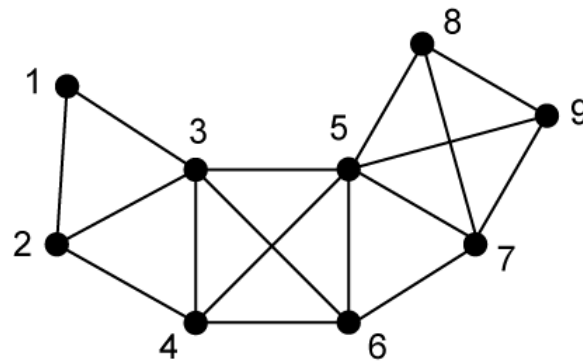


# Yoshimura-Kuh Channel Routing

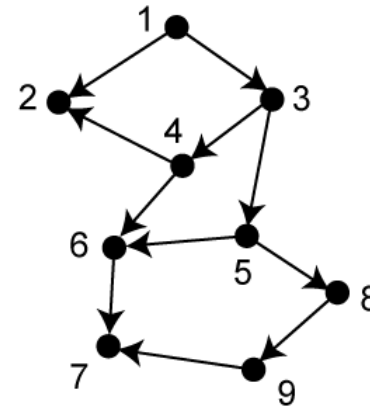
- Perform YK channel routing with  $K = 100$

TOP = [1,1,4,2,3,4,3,6,5,8,5,9]

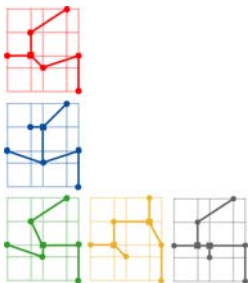
BOT = [2,3,2,0,5,6,4,7,6,9,8,7]



HCG

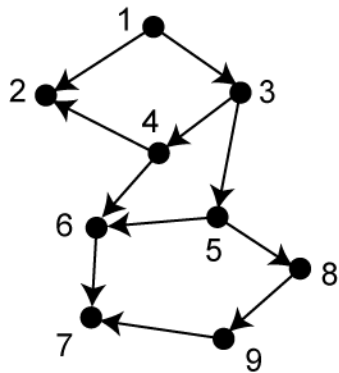


VCG

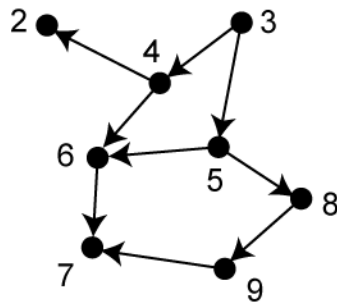


# Constrained Left-Edge Algorithm

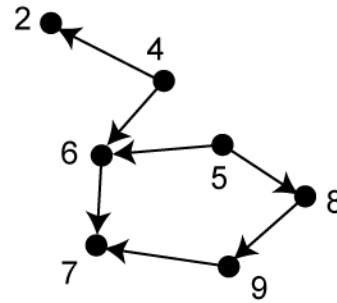
- First perform CLE on original problem (for comparison)
  - Assign VCG nodes with no incoming edge first
  - Use tracks top-to-bottom, left-to-right



(a)

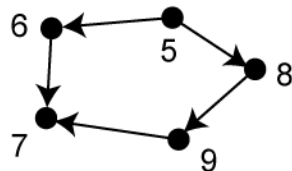


(b)

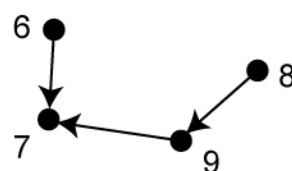


(c)

2 ●



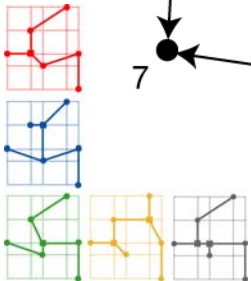
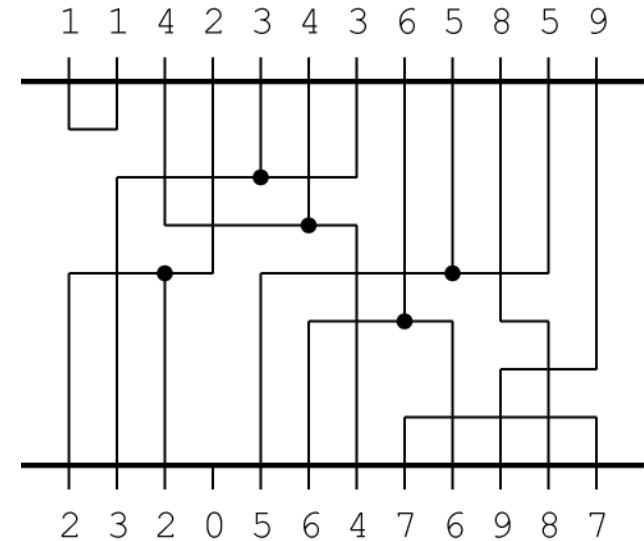
(d)



(e)



(f)



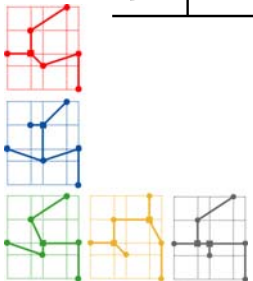
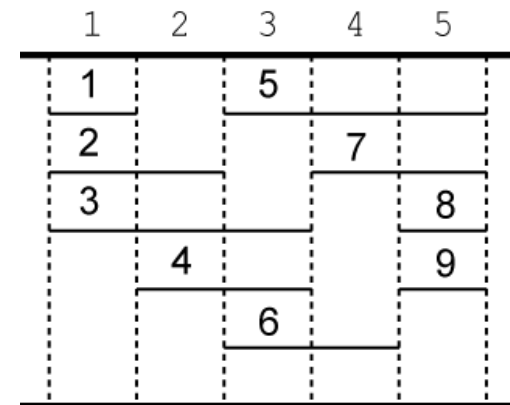
# Zone Representation

- Horizontal span of the nets and their zones

TOP = [1,1,4,2,3,4,3,6,5,8,5,9]

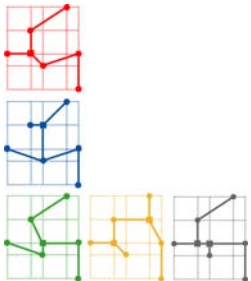
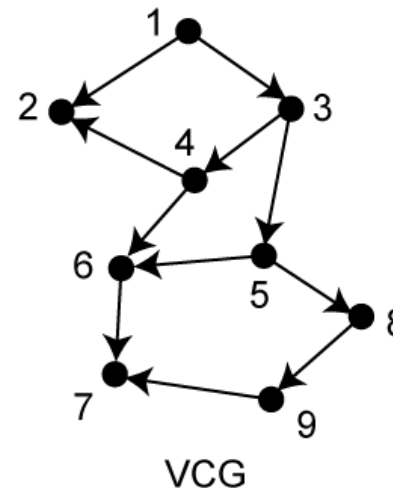
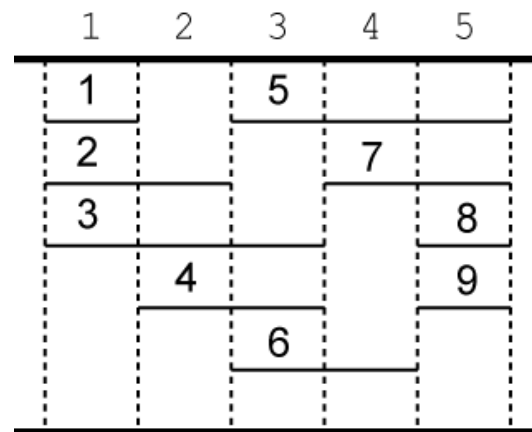
BOT = [2,3,2,0,5,6,4,7,6,9,8,7]

net	zone 1		zone 2		zone 3			zone 4		zone 5		
	c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	c11	c12
1	1	1										
2	2	2	2	2								
3		3	3	3	3	3	3					
4			4	4	4	4	4					
5					5	5	5	5	5	5		
6						6	6	6	6			
7								7	7	7	7	7
8										8	8	
9										9	9	9



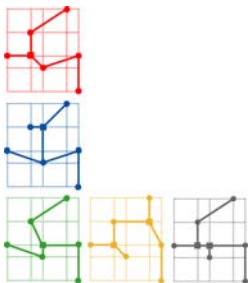
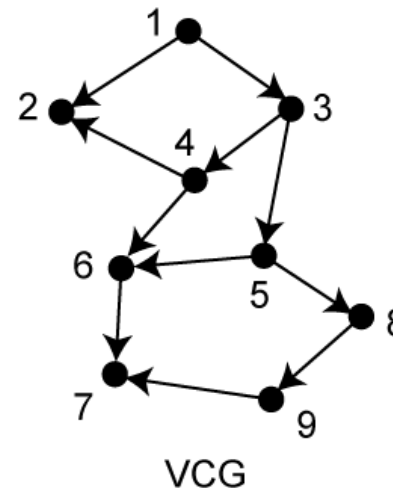
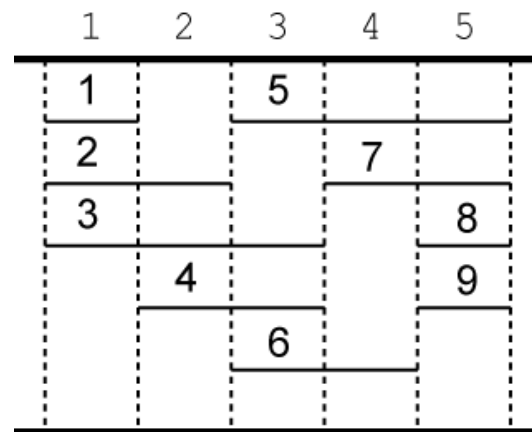
# Net Merging: Zone 1 and 2

- We compute
  - $L = \{1\}$  and  $R = \{4\}$
  - Net 1 and 4 are on the same path in VCG: no merging possible



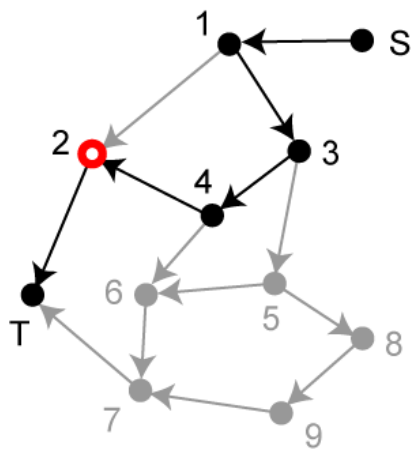
# Net Merging: Zone 2 and 3

- We compute
  - $L = \{1,2\}$  and  $R = \{5,6\}$  (= net 1 inherited from last step)
  - Merge-able pairs: (2,5) and (2,6) (= not on the same path in VCG)

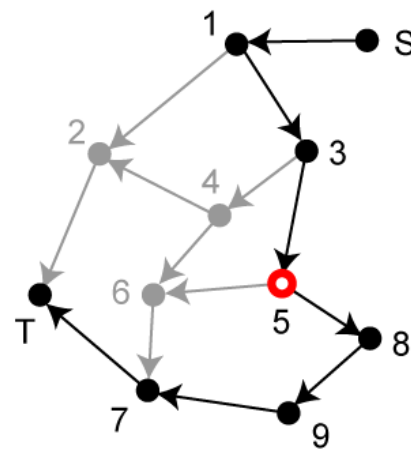


# Net Merging: Zone 2 and 3 (cont)

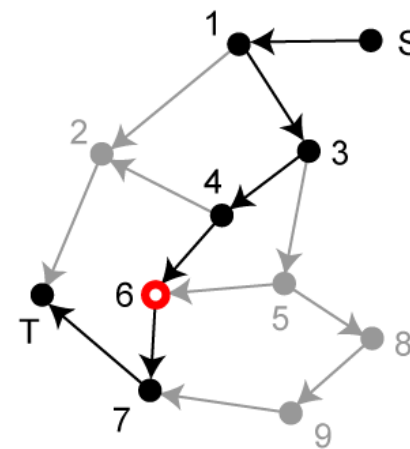
- Choose the “best” pair between (2,5) and (2,6)
  - We form  $P = \{5,6\}$  and  $Q = \{2\}$  and choose best from each set
  - We compute
    - $u(2) = 4, d(2) = 1, u(5) = 3, d(5) = 4, u(6) = 4, d(6) = 2$
  - Only 1 element in  $Q$ , so  $m^* = \text{net } 2$  trivially



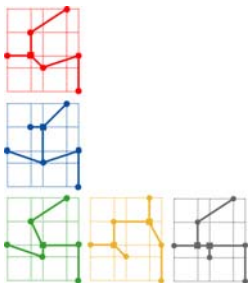
(a)



(b)



(c)





# Net Merging: Zone 2 and 3 (cont)

- Now choose “best” from  $P$

- We compute  $g(5,2)$  and  $g(6,2)$  using  $K = 100$

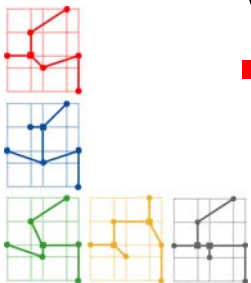
$$h(5, 2) = \max\{u(5), u(2)\} + \max\{d(5), d(2)\} \\ - \max\{u(5) + d(5), u(2) + d(2)\} = 1$$

$$h(6, 2) = \max\{u(6), u(2)\} + \max\{d(6), d(2)\} \\ - \max\{u(6) + d(6), u(2) + d(2)\} = 0$$

$$g(5, 2) = 100 \cdot h(5, 2) - \{\sqrt{u(2) \cdot u(5)} + \sqrt{d(2) \cdot d(5)}\} \\ = 94.5$$

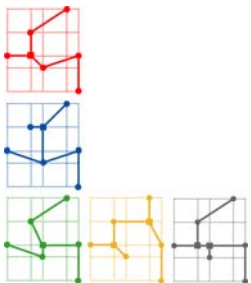
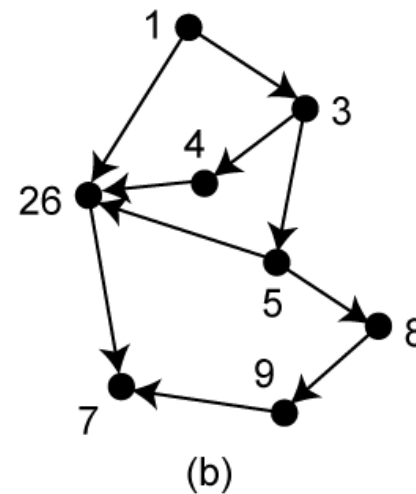
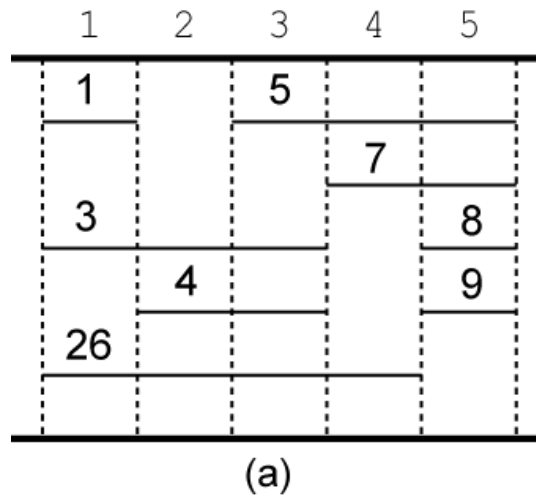
$$g(6, 2) = 100 \cdot h(6, 2) - \{\sqrt{u(2) \cdot u(6)} + \sqrt{d(2) \cdot d(6)}\} \\ = -5.4$$

- Since  $g(5,2) > g(6,2)$ , we choose  $n^* = \text{net } 6$
- We merge  $m^* = 2$  and  $n^* = 6$ 
  - **Likely** to minimize the increase in the longest path length in VCG



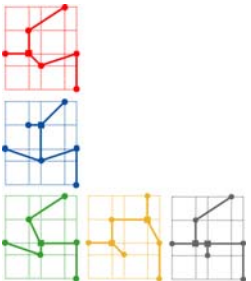
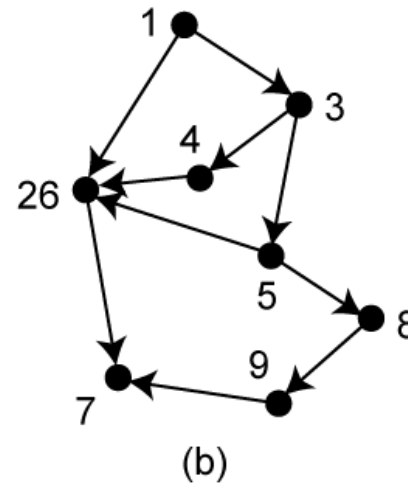
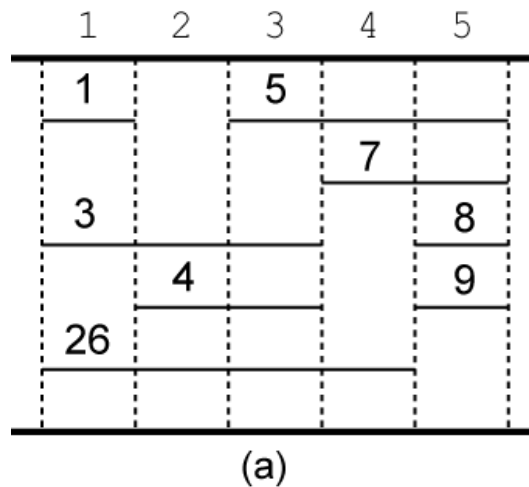
# Net Merging: Zone 2 and 3 (cont)

- Merged net 2 and 6
  - We had  $P = \{5,6\}$  and  $Q = \{2\}$ , and need to remove 2 and 6
    - $Q$  is empty, so we are done with zone 2 and 3
  - We had  $L = \{1,2\}$  and  $R = \{5,6\}$ , and need to remove 2 and 6
    - We keep  $L = \{1\}$
  - Updated zone representation and VCG



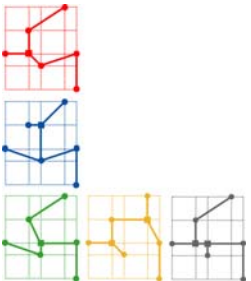
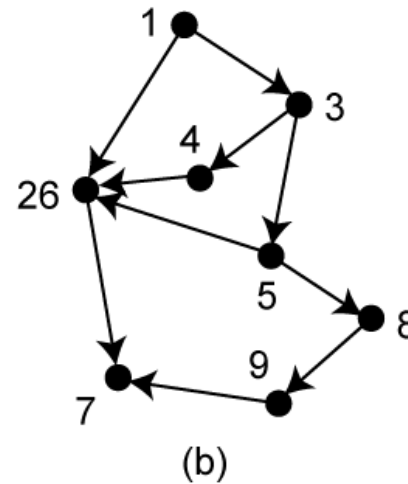
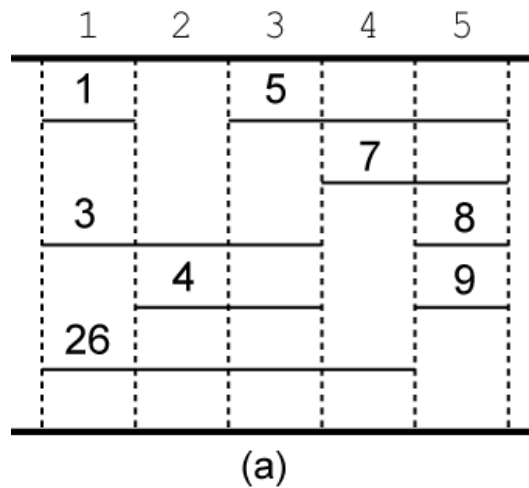
# Net Merging: Zone 3 and 4

- We compute
  - $L = \{1,3,4\}$  and  $R = \{7\}$  (= net 1 inherited from last step)
  - All nets in  $L$  and  $R$  are on the same path in VCG
    - no merging possible



# Net Merging: Zone 4 and 5

- We compute
  - $L = \{1,3,4,26\}$  and  $R = \{8,9\}$
  - Merge-able pairs:  $(4,8)$ ,  $(4,9)$ ,  $(26,8)$ ,  $(26,9)$



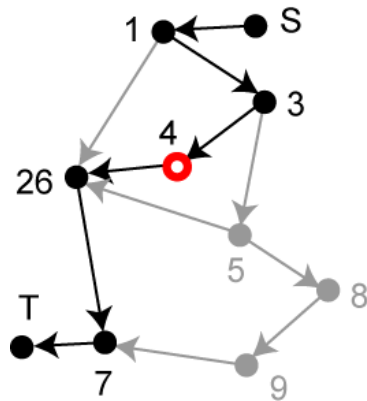
# Net Merging: Zone 4 and 5 (cont)

- Choose  $m^*$  from  $Q$

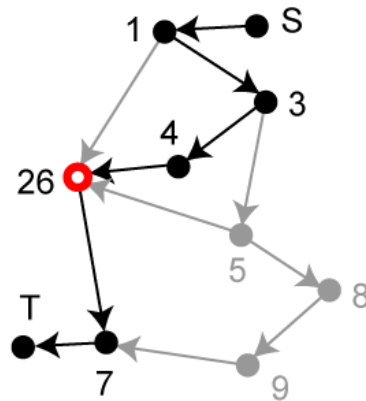
- We form  $P = \{4, 26\}$  and  $Q = \{8, 9\}$

- We compute

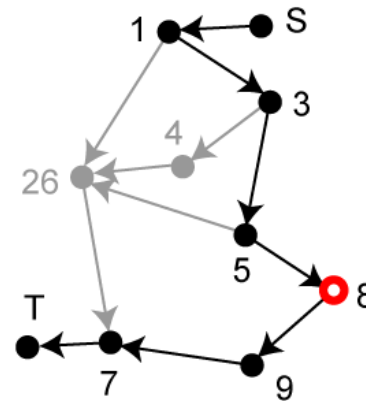
- $u(4) = 3, d(4) = 3, u(26) = 4, d(26) = 2, u(8) = 4, d(8) = 3, u(9) = 5, d(9) = 2$



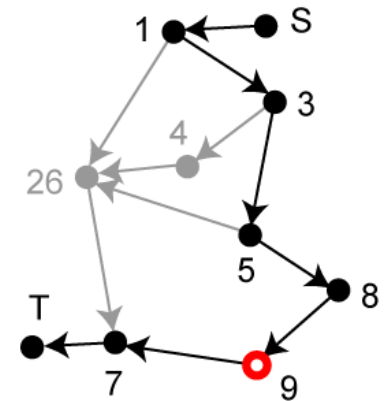
(a)



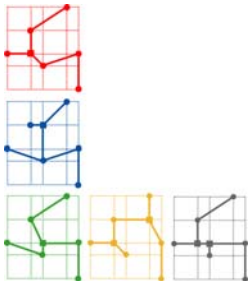
(b)



(c)

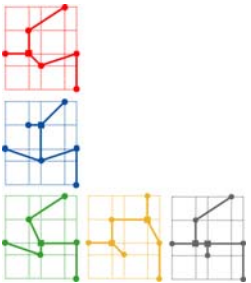


(d)



# Net Merging: Zone 4 and 5 (cont)

- Choose  $m^*$  from  $Q$  (cont)
  - We find  $m^*$  from  $Q$  that maximizes
    - $f(8) = 100 \cdot \{u(8) + d(8)\} + \max\{u(8), d(8)\} = 704$
    - $f(9) = 100 \cdot \{u(9) + d(9)\} + \max\{u(9), d(9)\} = 705$
  - So,  $m^* = 9$



# Net Merging: Zone 4 and 5 (cont)

- Choose  $n^*$  from  $P$

- We compute  $g(4,9)$  and  $g(26,9)$  using  $K = 100$

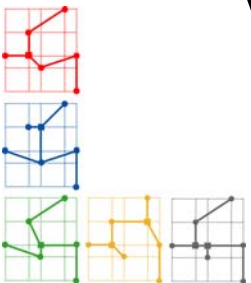
$$h(4,9) = \max\{u(4), u(9)\} + \max\{d(4), d(9)\} \\ - \max\{u(4) + d(4), u(9) + d(9)\} = 1$$

$$h(26,9) = \max\{u(26), u(9)\} + \max\{d(26), d(9)\} \\ - \max\{u(26) + d(26), u(9) + d(9)\} = 0$$

$$g(4,9) = 100 \cdot h(4,9) - \{\sqrt{u(9) \cdot u(4)} + \sqrt{d(9) \cdot d(4)}\} \\ = 93.7$$

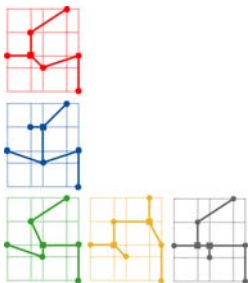
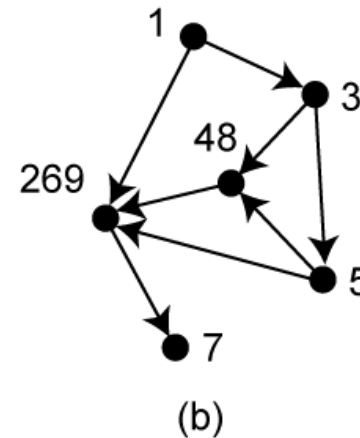
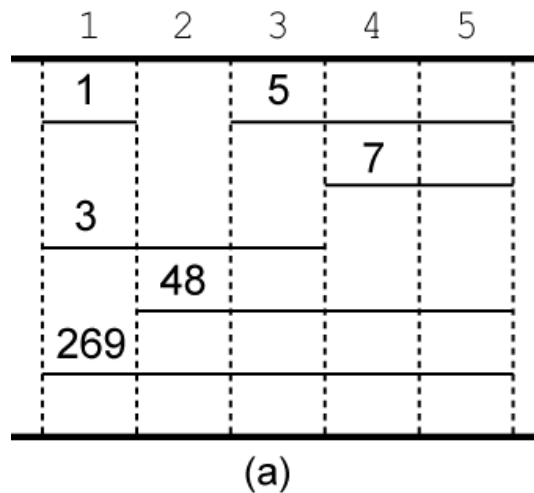
$$g(26,9) = 100 \cdot h(26,9) - \{\sqrt{u(9) \cdot u(26)} + \sqrt{d(9) \cdot d(26)}\} \\ = -6.5$$

- Since  $g(4,9) > g(26,9)$ , we get  $n^* = \text{net } 26$
- We merge  $m^* = 9$  and  $n^* = 26$



# Net Merging: Zone 4 and 5 (cont)

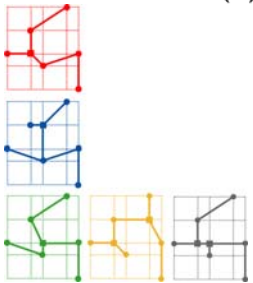
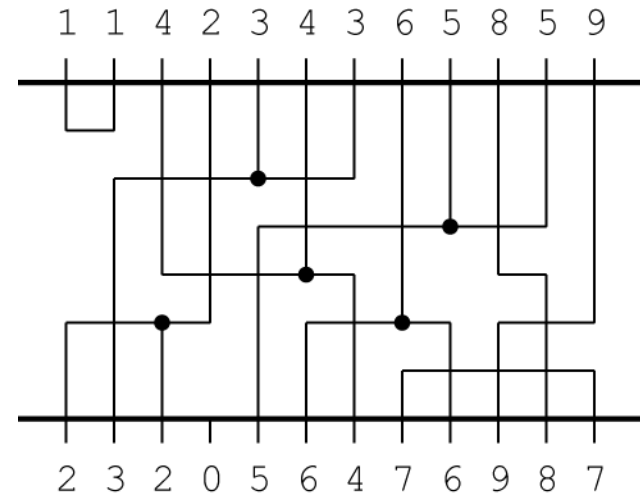
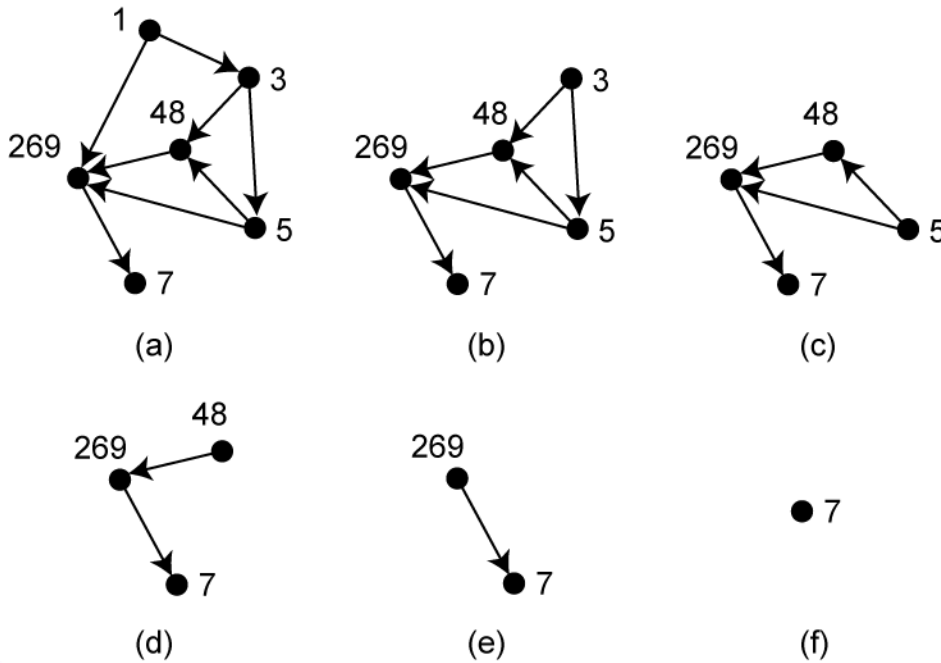
- Merged net 26 and 9
  - We had  $P = \{4,26\}$  and  $Q = \{8,9\}$ , and need to remove 26 and 9
    - $Q$  is not empty, so we **repeat** the whole process
  - Updated  $P = \{4\}$  and  $Q = \{8\}$ 
    - Trivial to see that  $m^* = 8$  and  $n^* = 4$ , so we merge 8 and 4
  - Updated zone representation and VCG





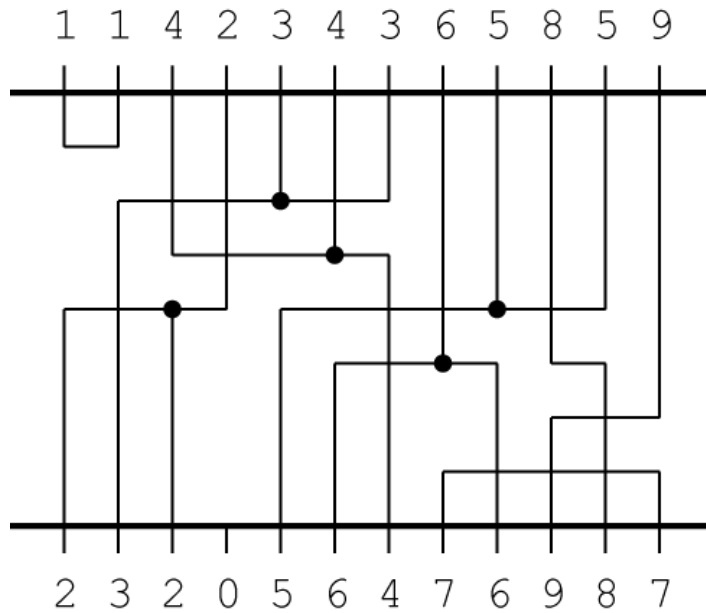
# Routing with Merged Nets

- Perform CLE on merged netlist
  - Use tracks top-to-bottom, left-to-right

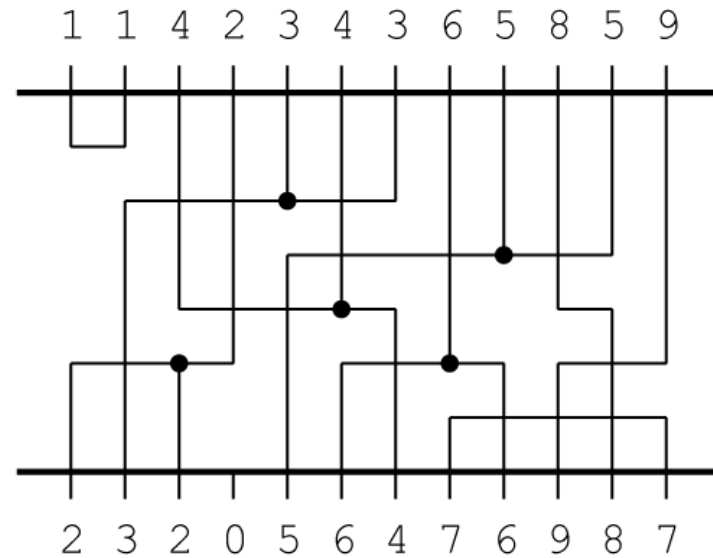


# Comparison

- Net merging helped
  - Reduce channel height by 1



without net merging



with net merging

