Steiner Routing

ECE6133

Physical Design Automation of VLSI Systems

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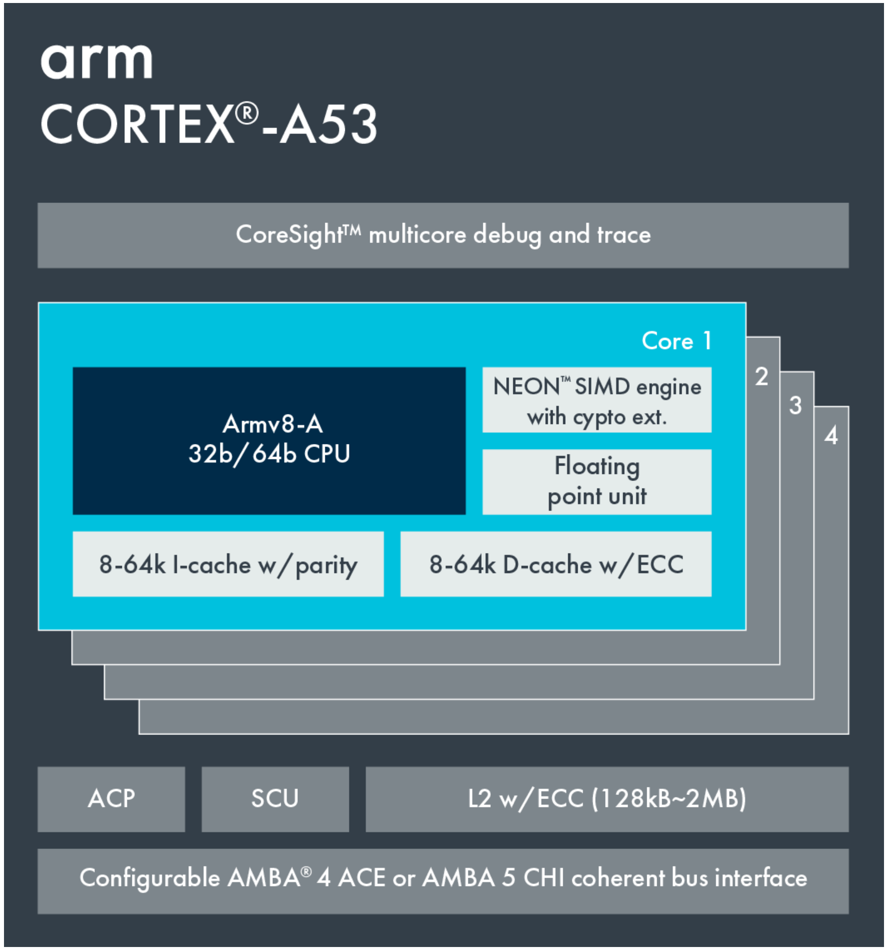
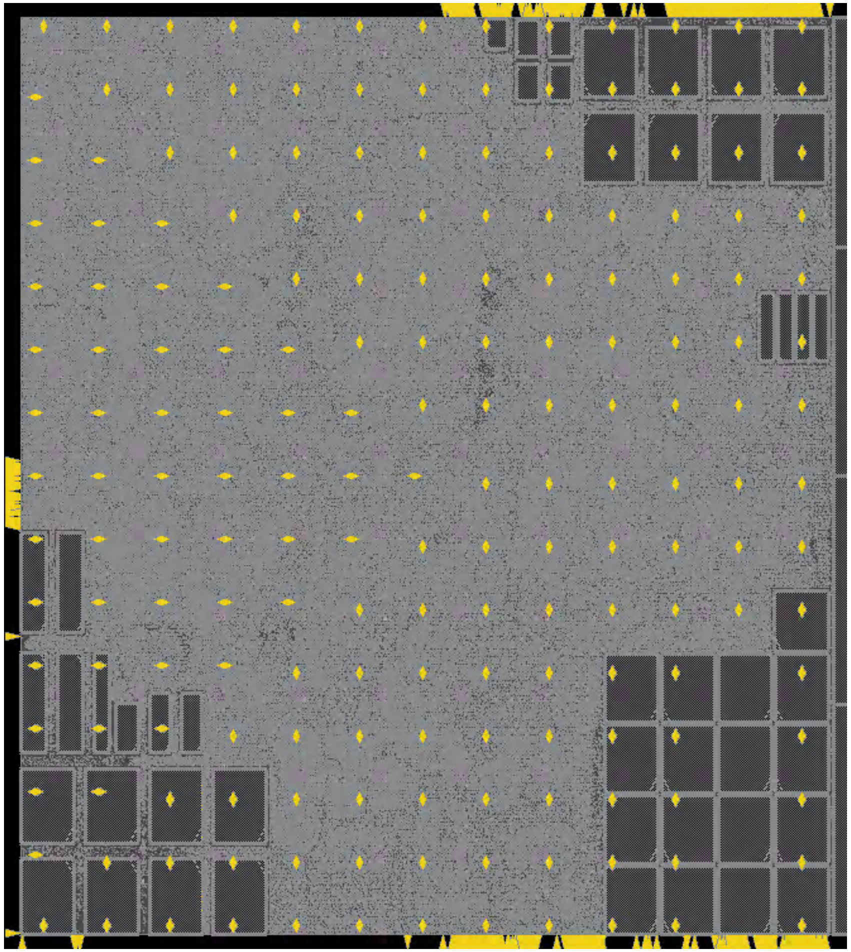
ARM A53 Placement



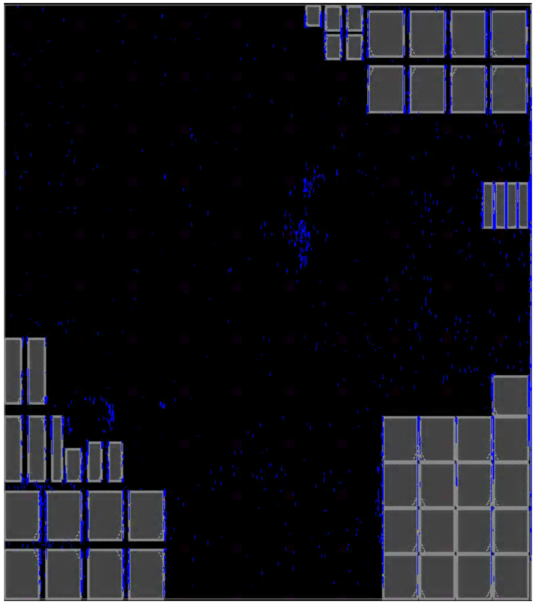


TSMC 28nm BEOL Spec

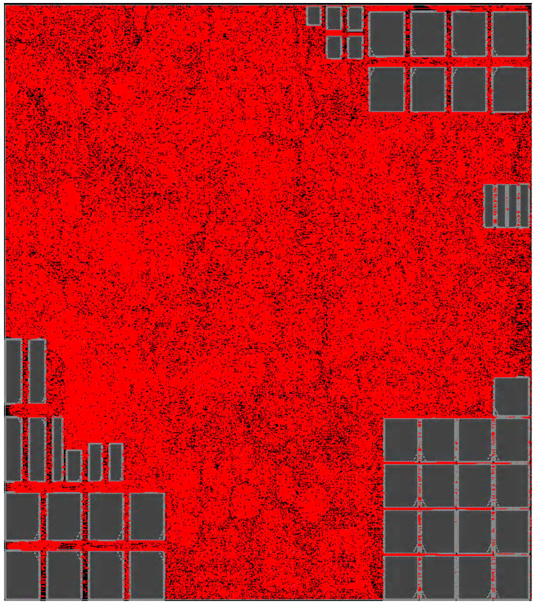
	Width (um)	Pitch (um)	Dir.
M1	0.05	0.135	V
M2	0.05	0.100	H
M3	0.05	0.100	V
M4	0.05	0.100	H
M5	0.05	0.100	V
M6	0.05	0.100	H

	R (ohm/um)	C (fF/um)
M1	7.24	0.172
M2	9.05	0.175
M3	9.06	0.181
M4	9.05	0.177
M5	9.06	0.180
M6	9.05	0.177

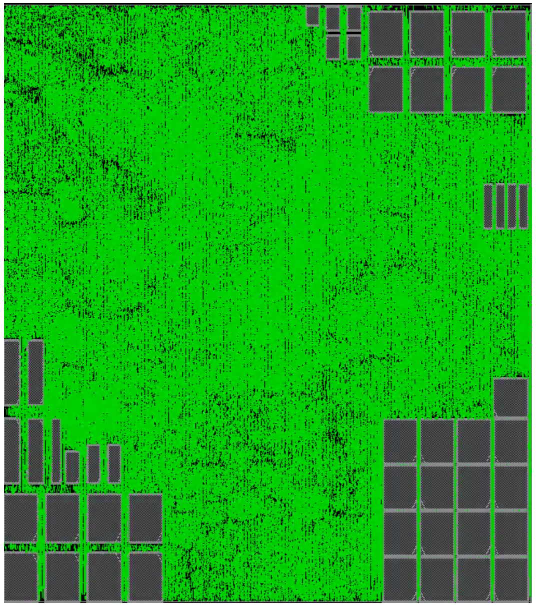
Full-Chip Routing



M1

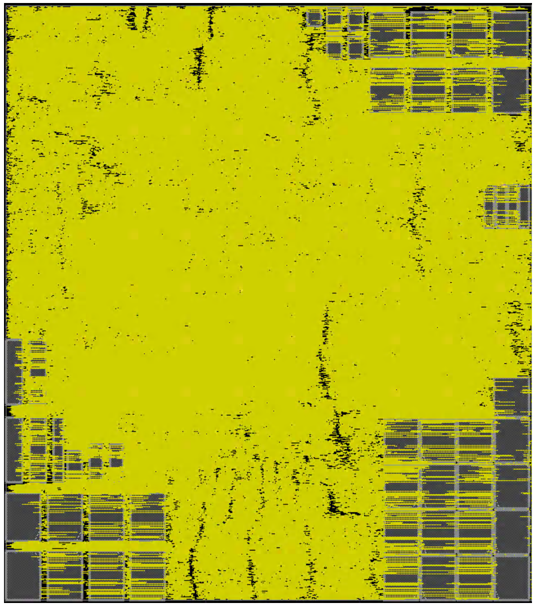


M2



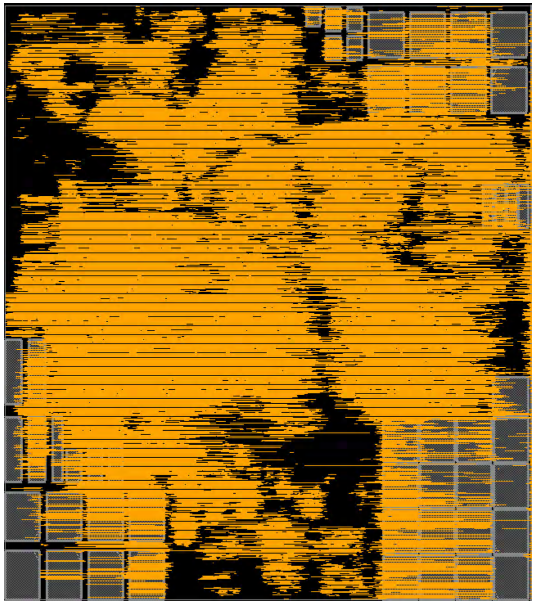
M3

Full-Chip Routing



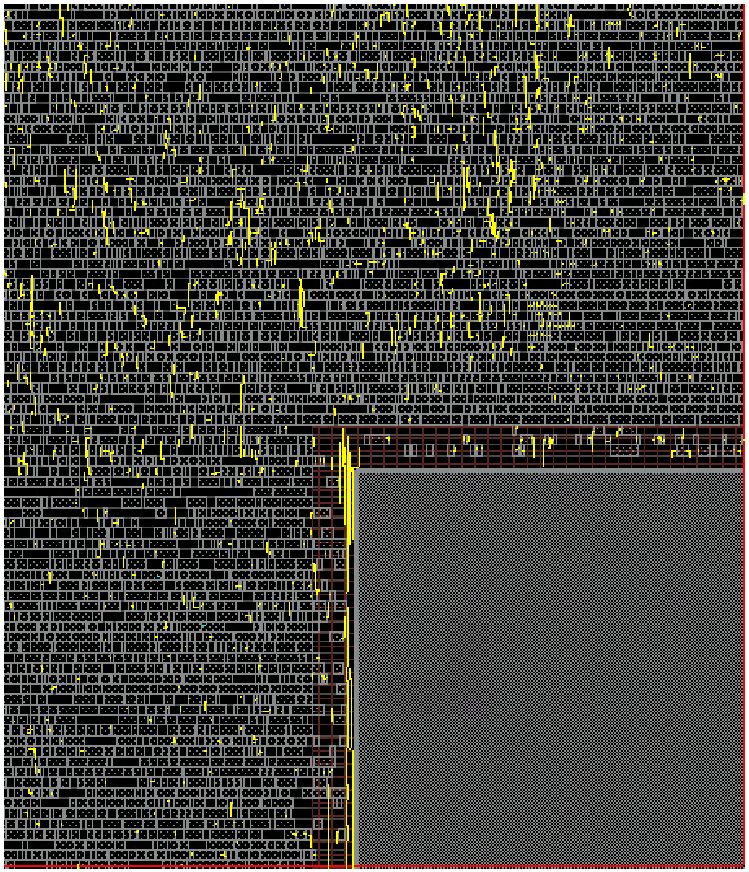
M4

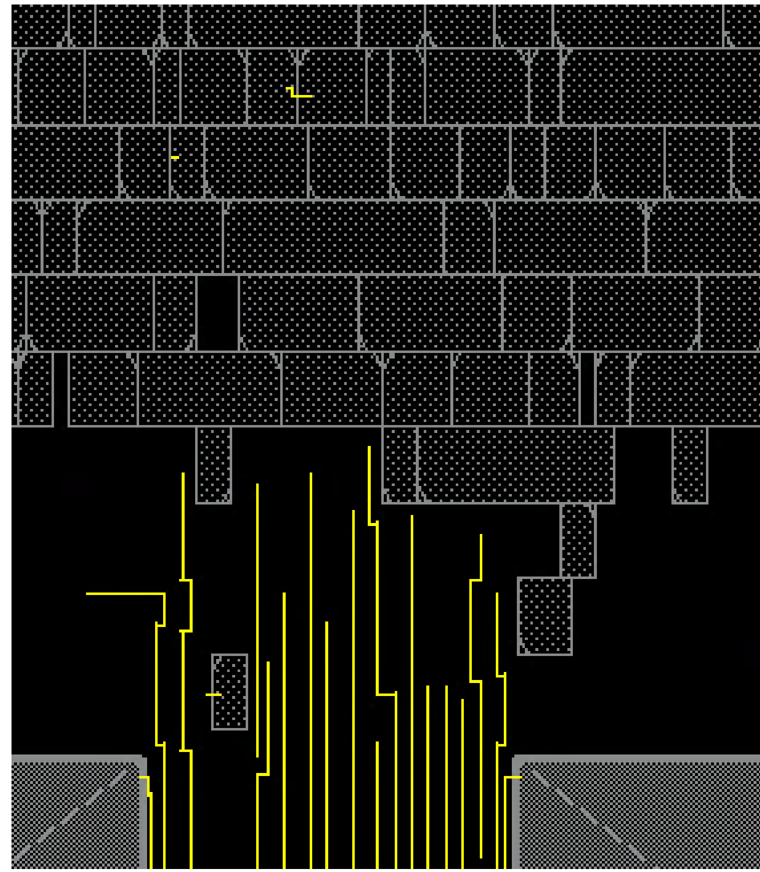
M5



M6

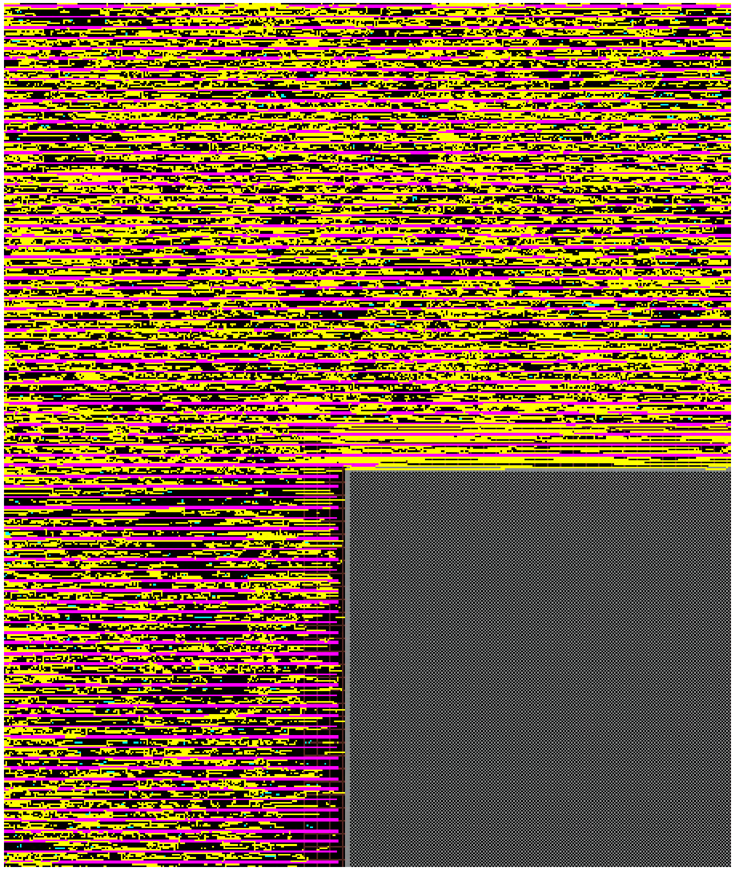
M1 Layer (Mostly Intra-Cell Routing)

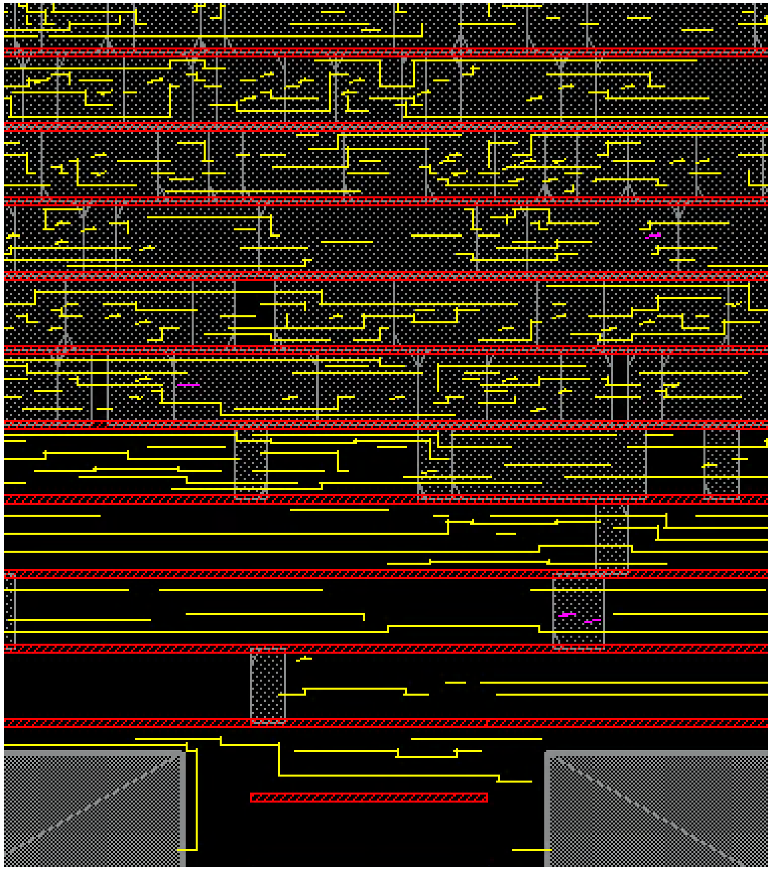




yellow: signal

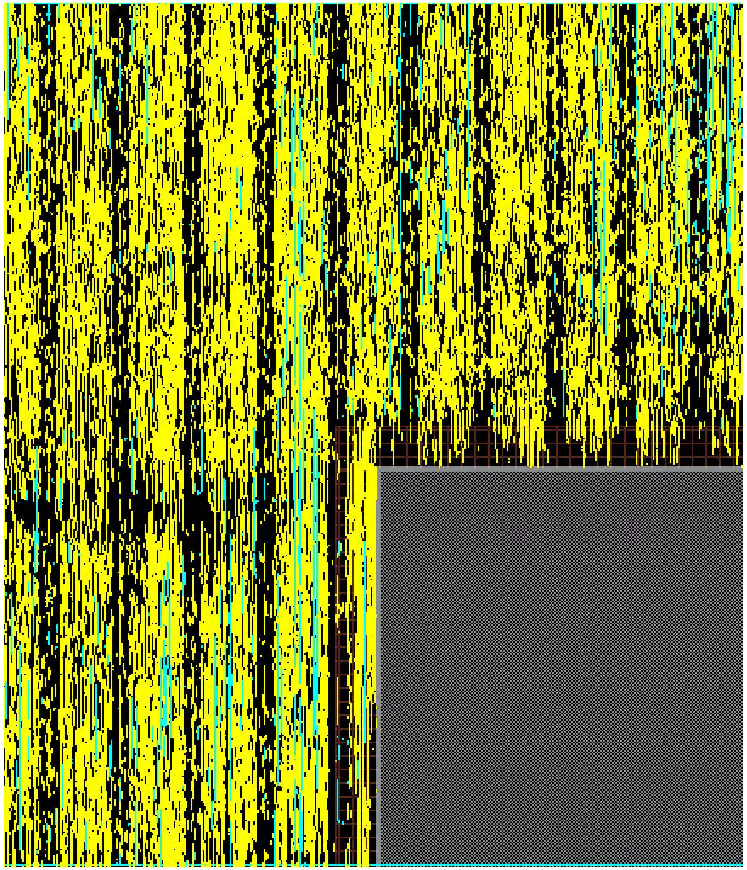
M2 Layer

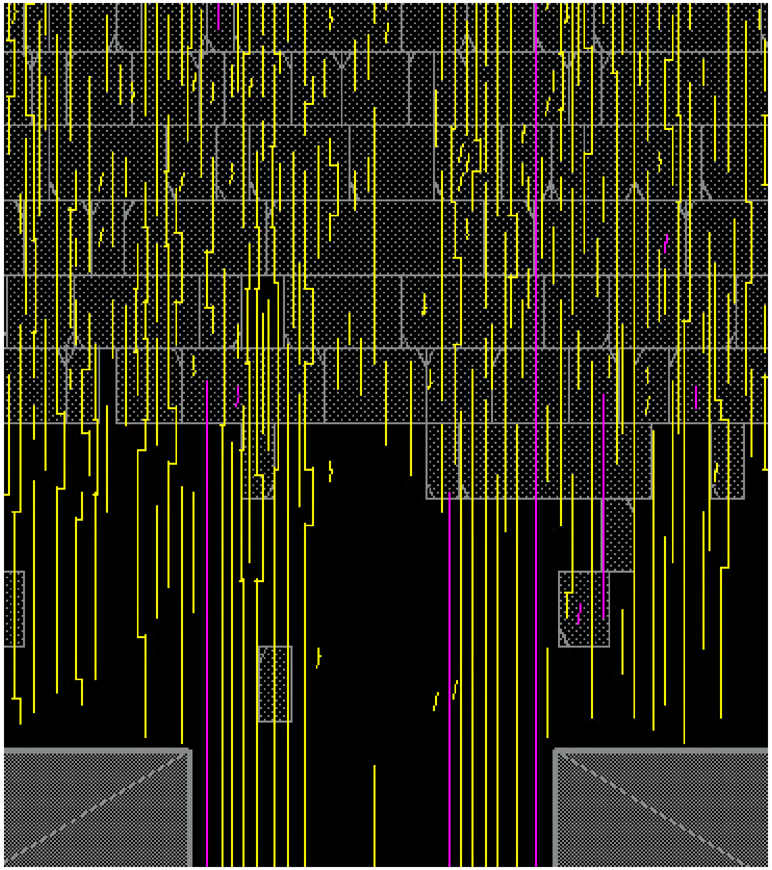




yellow: signal
magenta: clock, red: power/ground

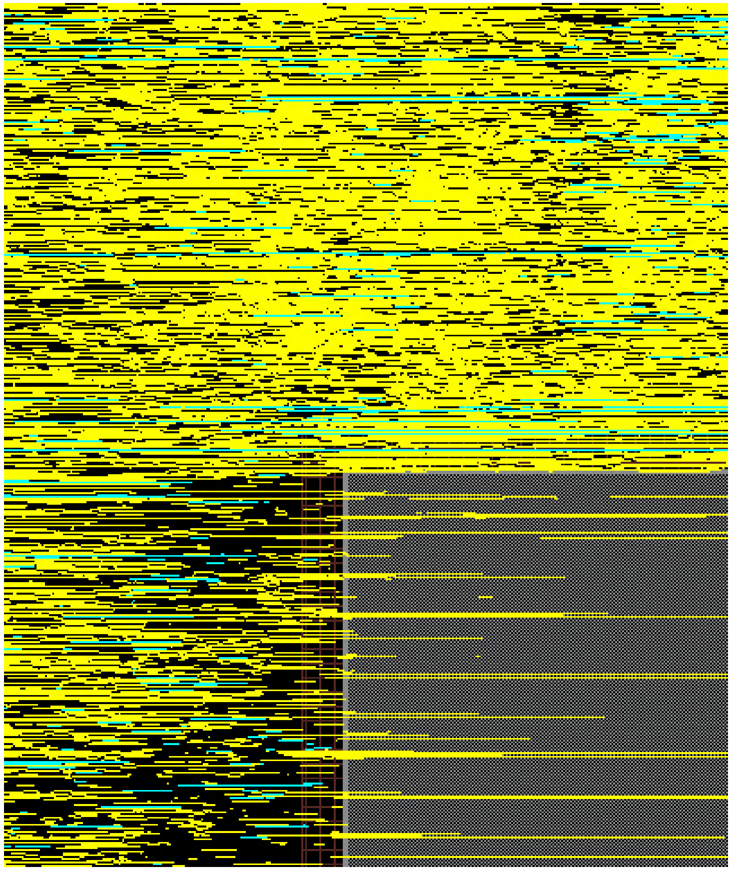
M3 Layer

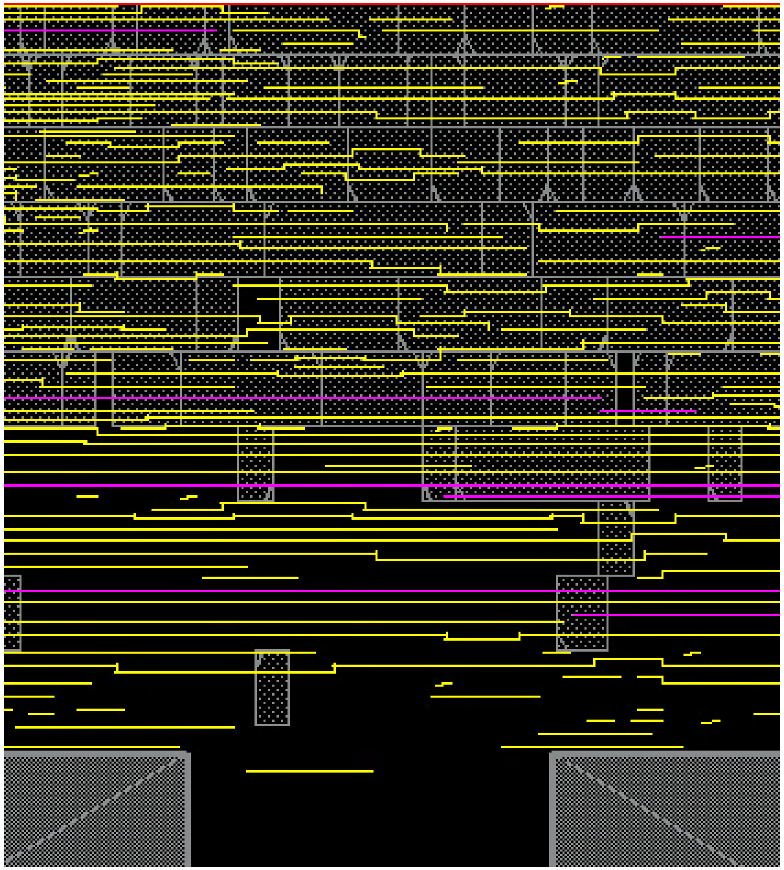




yellow: signal
magenta: clock

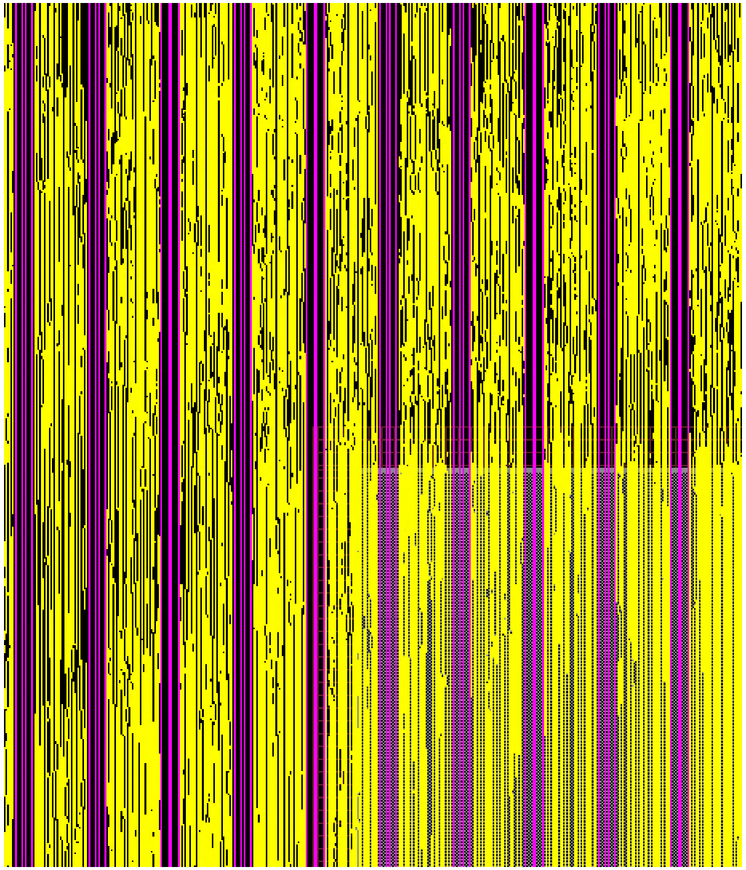
M4

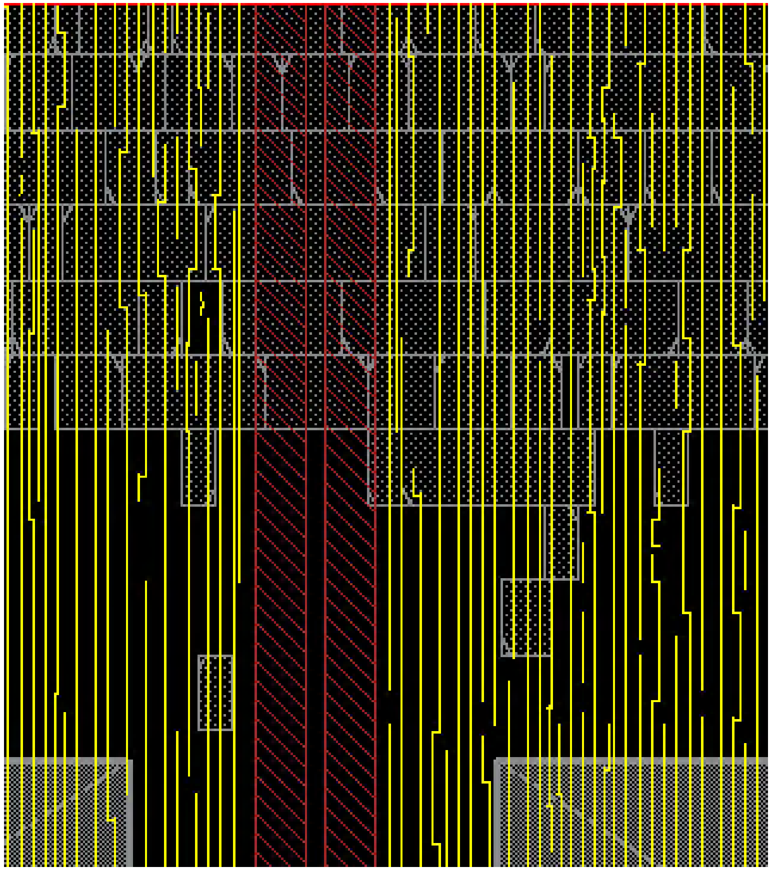




yellow: signal
magenta: clock

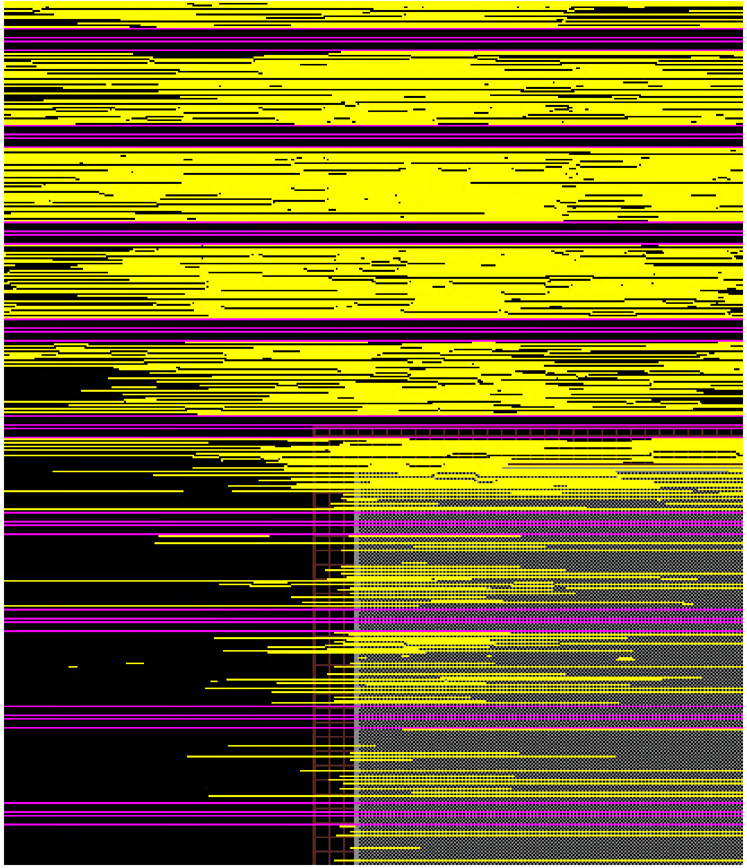
M5

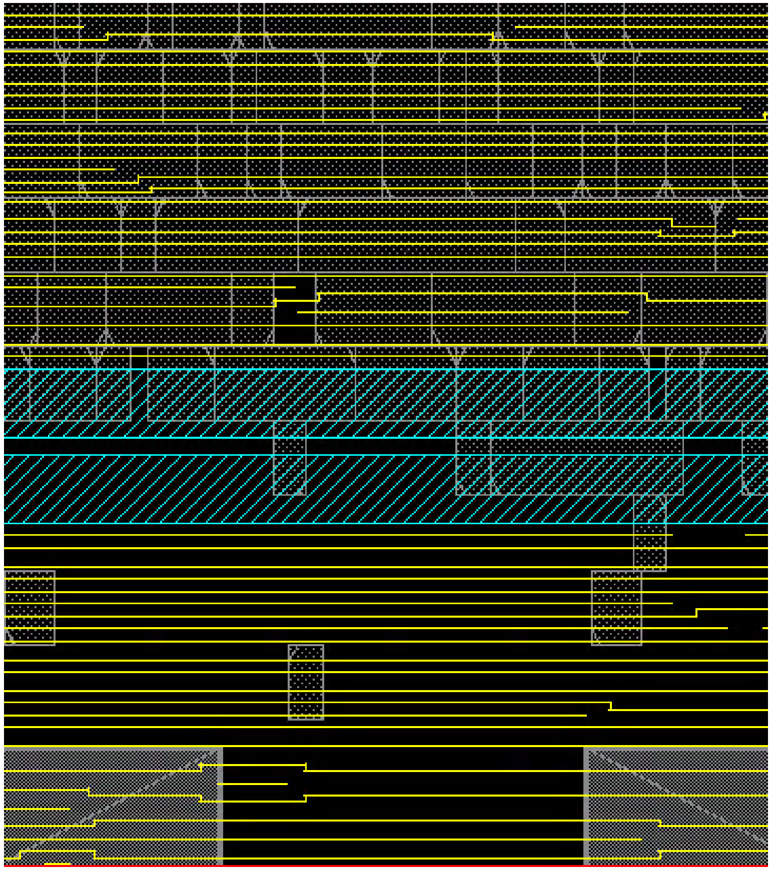




yellow: signal
magenta: clock, red: power/ground

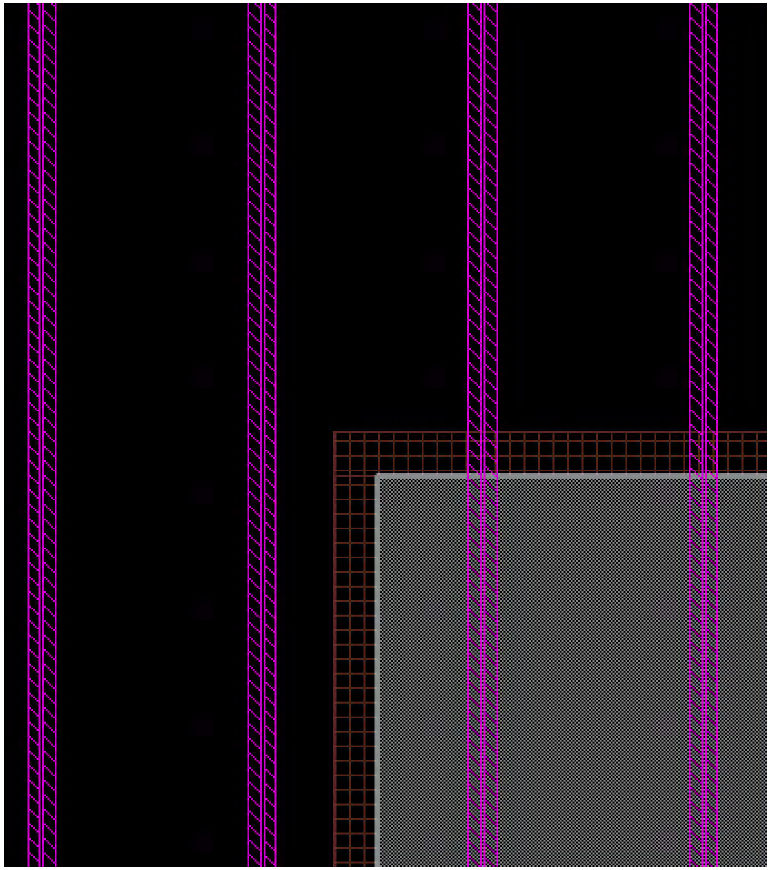
M6

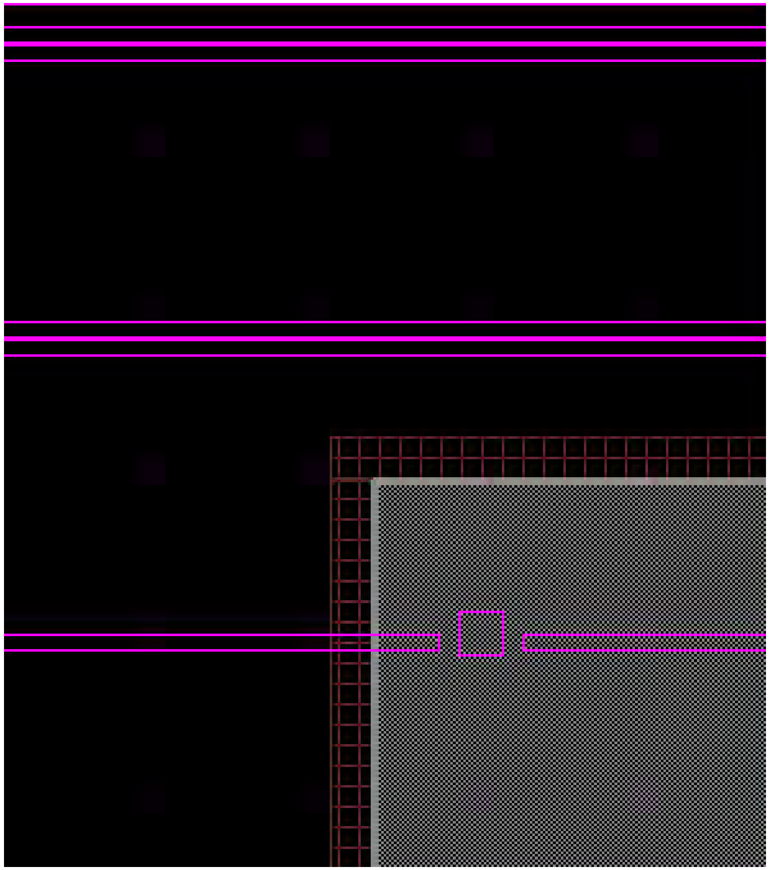




yellow: signal
cyan: power/ground

M7 and M8





magenta: power/ground

Routing

placement

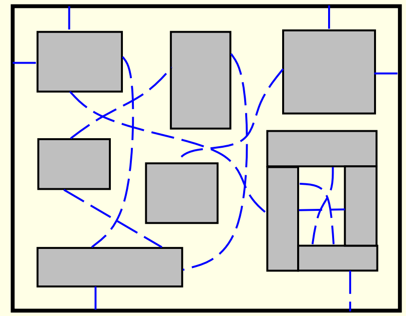
- Generates a "loose" route for each net.
- Assigns a list of routing regions to each net without specifying the actual layout of wires.

global routing

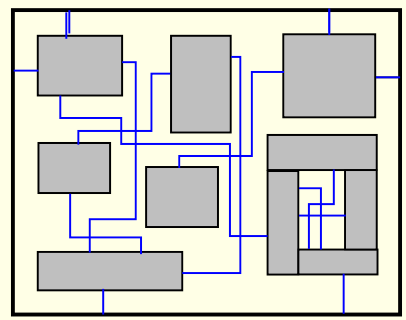
detailed routing

- Finds the actual geometric layout of each net within the assigned routing regions.

compaction



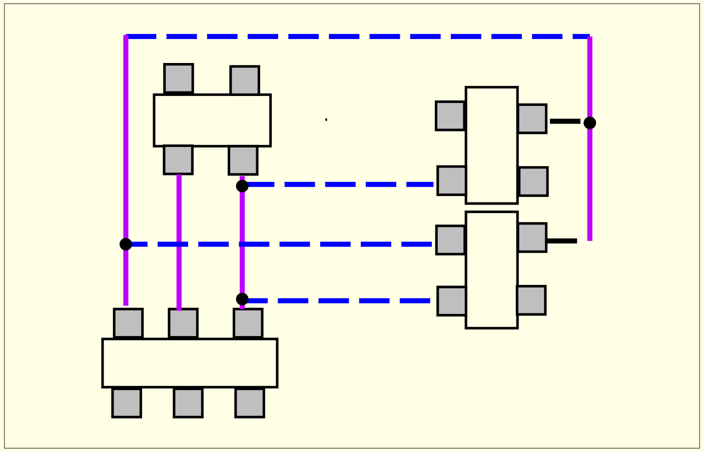
Global routing



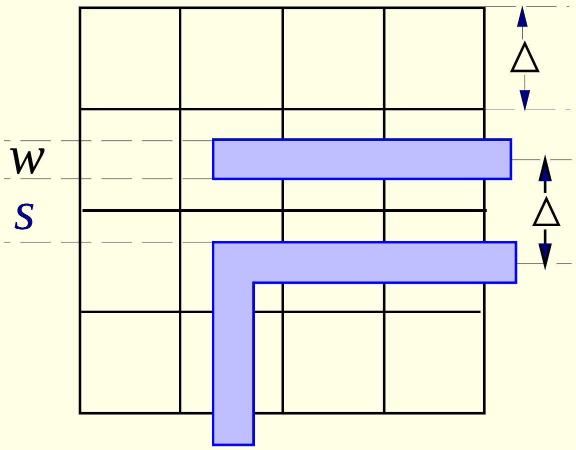
Detailed routing

Routing Constraints

- 100% routing completion + area minimization, under a set of constraints:
  - Placement constraint: usually based on fixed placement
  - Number of routing layers
  - Geometrical constraints: must satisfy design rules
  - Timing constraints (performance-driven routing): must satisfy delay constraints
  - Crosstalk?
  - Process variations?



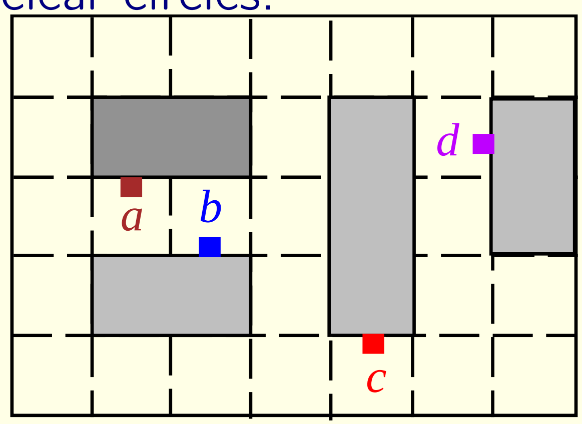
Two−layer routing

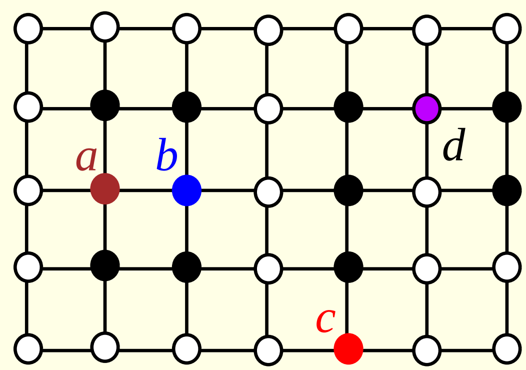


Geometrical constraint

Graph Models for Global Routing: Grid Graph

- Each cell is represented by a vertex.
- Two vertices are joined by an edge if the corresponding cells are adjacent to each other.
- The occupied cells are represented as filled circles, whereas the others are as clear circles.



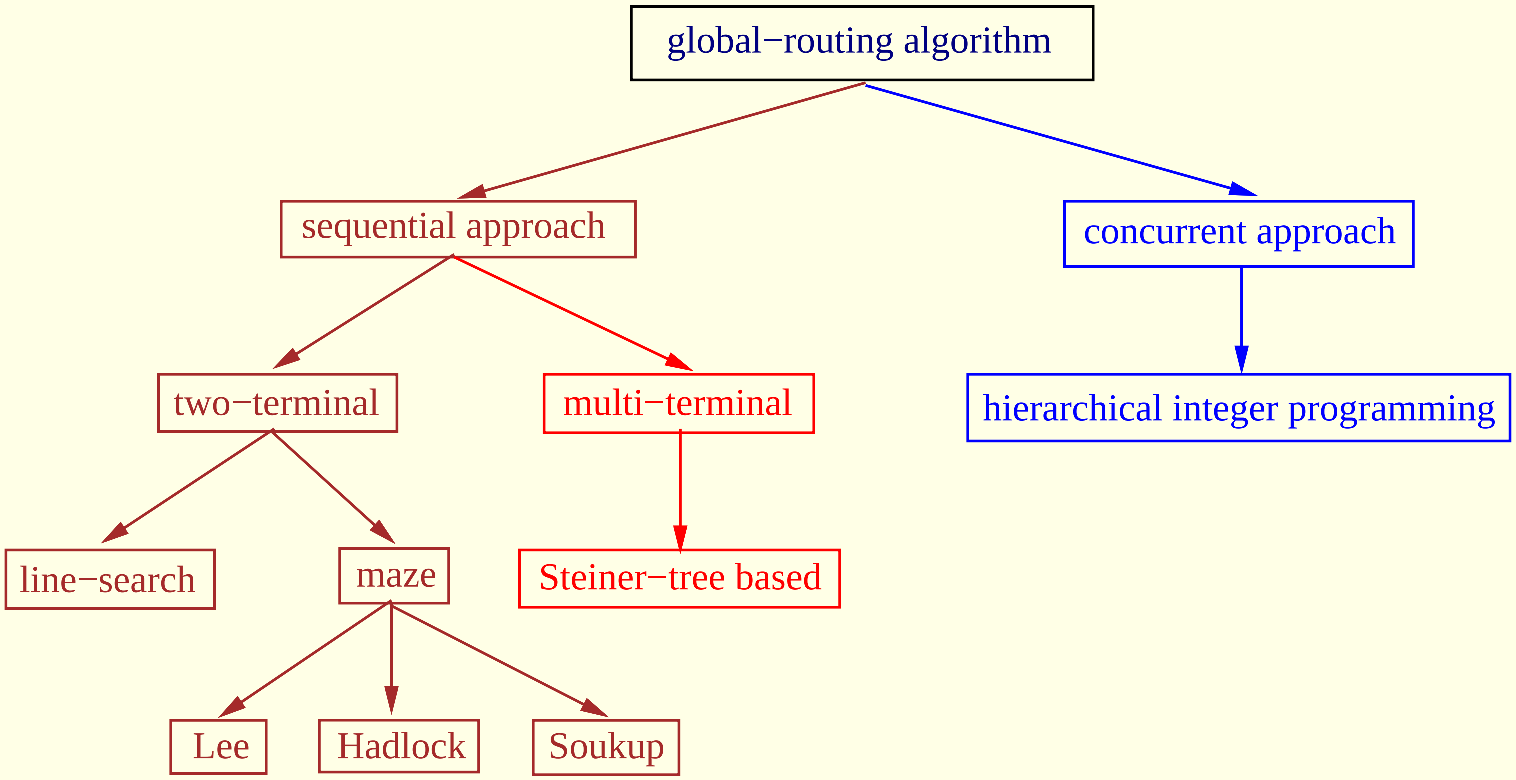


Global-Routing Problem

- Given a netlist N=$\{N_1, N_2, \ldots, N_n\}$, a routing graph $G = (V, E)$, find a Steiner tree T_i for each net N_i, $1 \leq i \leq n$, such that $U(e_j) \leq c(e_j)$, $\forall e_j \in E$ and $\sum_{i=1}^{n} L(T_i)$ is minimized,
  where
  - $c(e_j)$: capacity of edge e_j;
  - $x_{ij} = 1$ if e_j is in T_i; $x_{ij} = 0$ otherwise;
  - $U(e_j) = \sum_{i=1}^{n} x_{ij}$: # of wires that pass through the channel corresponding to edge e_j;
  - $L(T_i)$: total wirelength of Steiner tree T_i.
- For high-performance, the maximum wirelength ($\max_{i=1}^{n} L(T_i)$) is minimized (or the longest path between two points in T_i is minimized).

Classification of Global-Routing Algorithm

- **Sequential approach:** Assigns priority to nets; routes one net at a time based on its priority (net ordering?).
- **Concurrent approach:** All nets are considered at the same time (complexity?)



Spanning Tree

Problem Formulation:

Given a graph $G = (V, E)$, select a subset $V' \subseteq V$, such that V' has property $\mathcal{P}$.

Minimum Spanning Tree

Problem Formulation:

Given an edge-weighted graph $G = (V, E)$, select a subset of edges $E' \subseteq E$ such that E' induces a tree and the total cost of edges $\Sigma_{e_i \in E'} wt(e_i)$, is minimum over all such trees, where $wt(e_i)$ is the cost or weight of the edge e_i.

– Used in routing applications.

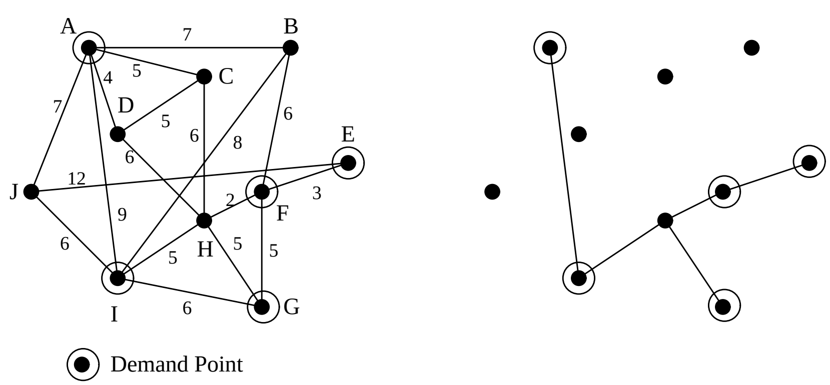
Steiner Trees

1. **Problem formulation:**

   Given an edge weighted graph $G = (V, E)$ and a subset $D \subseteq V$, select a subset $V' \subseteq V$, such that $D \subseteq V'$ and V' induces a tree of minimum cost over all such trees.

   The set D is referred to as the set of *demand points* and the set $V' - D$ is referred to as *Steiner points*.

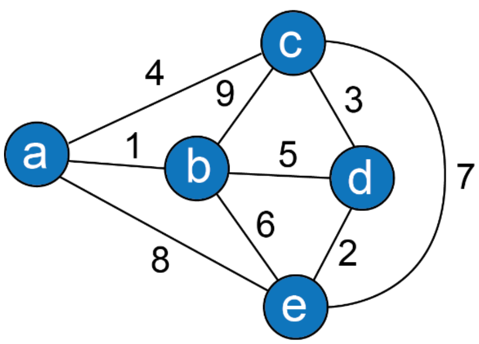
- Used in the global routing of multi-terminal nets.



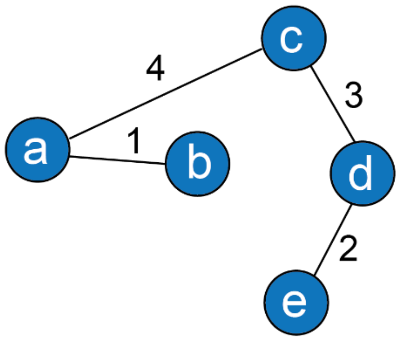
(a) (b)

Min Spanning Trees vs. Steiner Trees

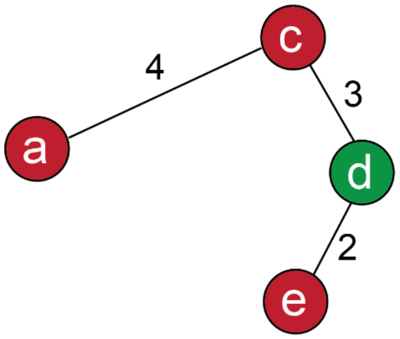
- Both problems try to “span” nodes in the given graph
  - Goal is to minimize the total edge weight
  - MST: span all nodes
  - Steiner tree: span only a designated subset of nodes. We can use “extra” nodes (= steiner nodes) if they help.



input graph



minimum spanning tree
total cost = 10

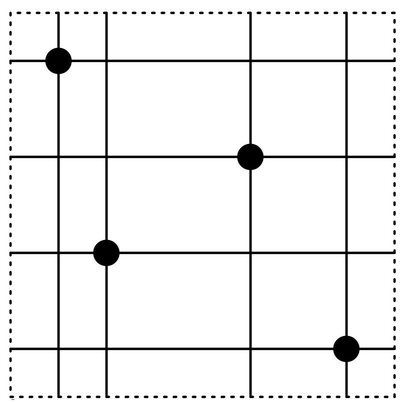


steiner tree for {a, c, e}
steiner point = {d}
total cost = 9

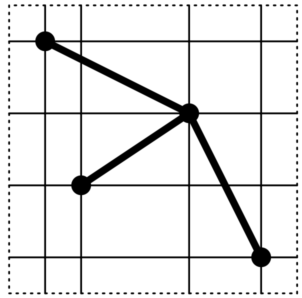
Underlying Grid Graph

The underlying grid graph is defined by the intersections of the horizontal and vertical lines drawn through the demand points.

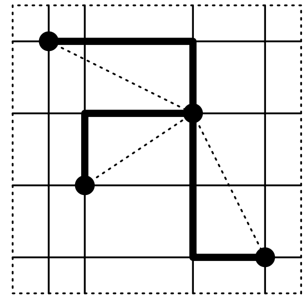
Hanan's Thm (69'): There exists an optimal RST with all Steiner points (set S) chosen from the intersection points of horizontal and vertical lines drawn from points of D.



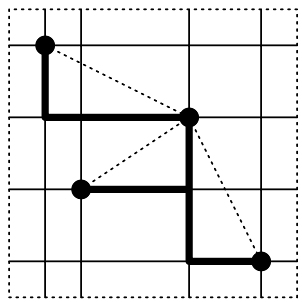
Different Steiner trees constructed from a MST



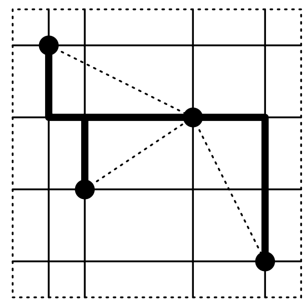
(a)



(b)

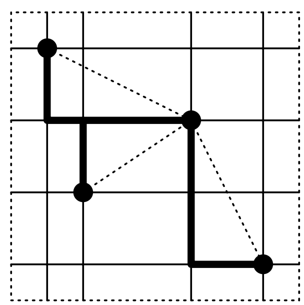


(c)



(d)

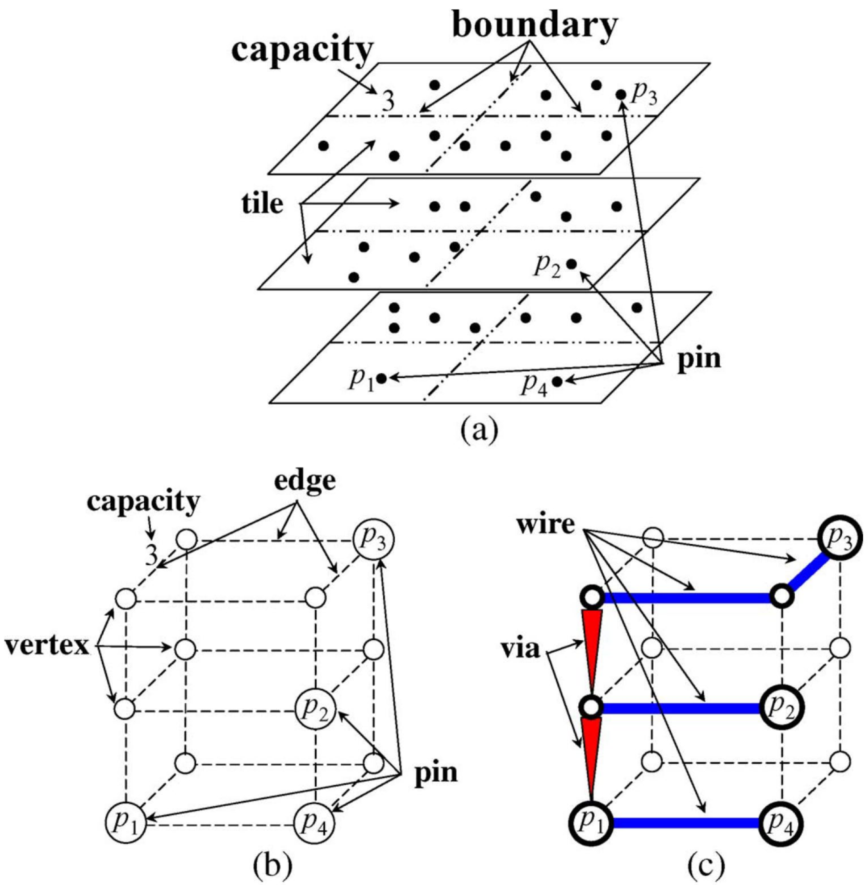
Hwang's Thm (76'):
The ratio of the cost
of a rectilinear MST
to that of an optimal
RST is no greater
than 3/2.



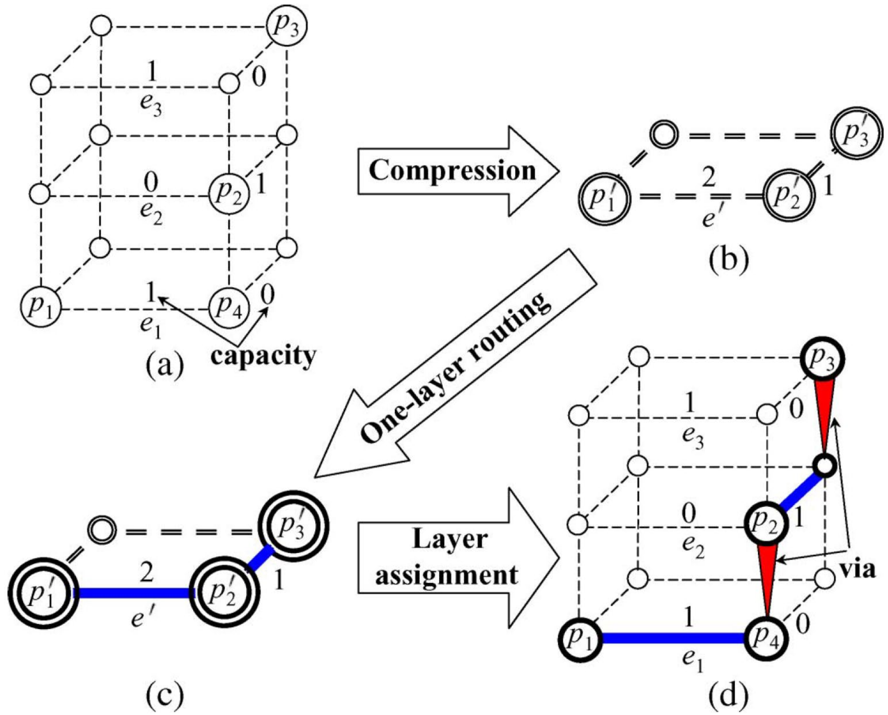
(e)

Steiner Routing: 3D vs. 2D

routing problem instance



3D Steiner Routing



2D Steiner Routing + Layer Assignment

The 1-Steiner Problem

- **Definition**

We denote the minimum spanning tree over a point set P by $MST(P)$, and use $c(MST(P))$ to denote the cost of the MST on point set P. Given a point set $P = \{p_1, \cdots, p_n\}$, a *1-Steiner point* is any point x such that $c(MST(P \cup \{x\}))$ is minimized, with $c(MST(P \cup \{x\})) < c(MST(P))$. A *1-Steiner tree* is the minimum spanning tree over $P \cup \{x\}$.

Why 1-Steiner Insertion?

- **Can Reduce Wirelength**

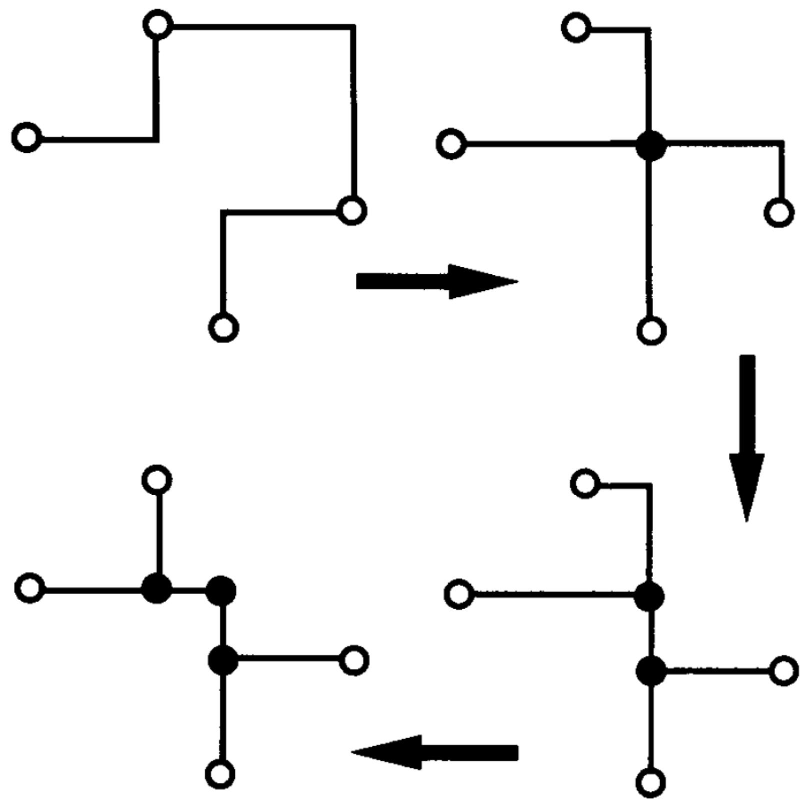


Fig. 3. Execution of iterated 1-Steiner on a four-point example.

1-Steiner by Kahng/Robins

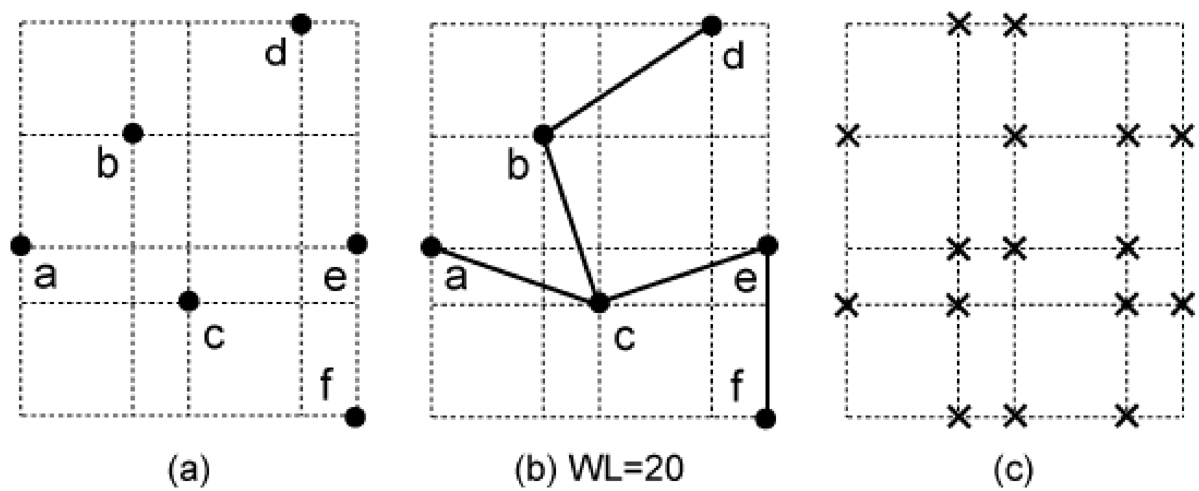
- **Iterative 1-Steiner Insertion Algorithm**
  - **Keep adding 1-Steiner point one-by-one until no more gain**

By the result of Hanan, we can find a 1-Steiner point by constructing a new MST on $n + 1$ points for each element in the Steiner candidate set, then picking the candidate which results in the shortest MST.

- **Naïve implementation: $O(n^2 \times n \log n \times n)$**
- **Sophisticated implementation: $O(n^3)$**

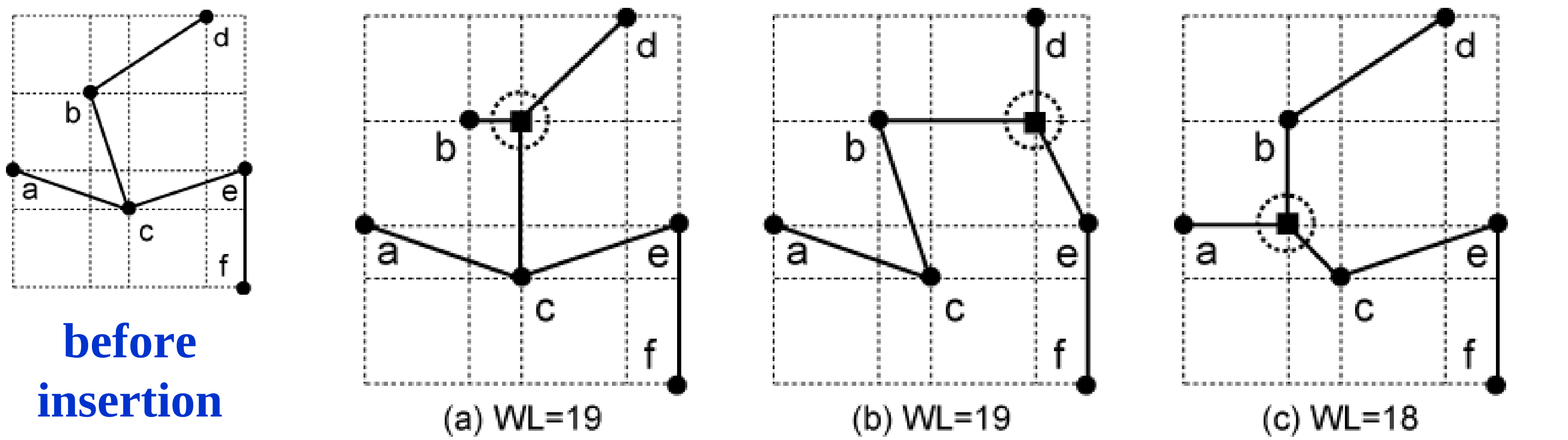
1-Steiner Routing by Kahng/Robins

- Perform 1-Steiner Routing by Kahng/Robins
  - Need an initial MST: wirelength is 20
  - 16 locations for Steiner points

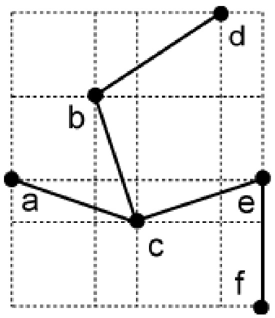


First 1-Steiner Point Insertion

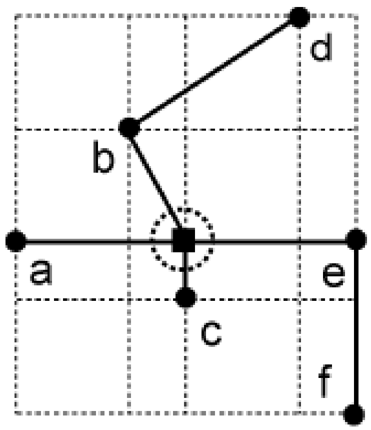
- There are six 1-Steiner points
  - Two best solutions: we choose (c) randomly



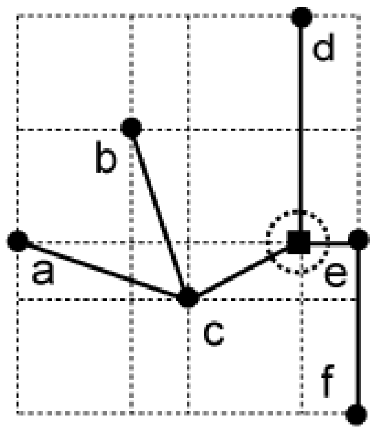
First 1-Steiner Point Insertion (cont)



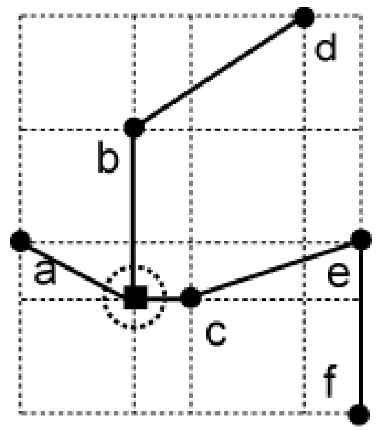
before insertion



(d) WL=18



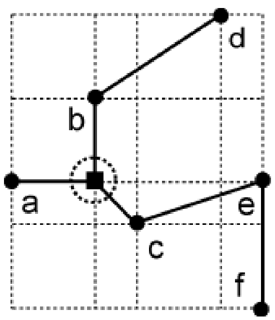
(e) WL=19



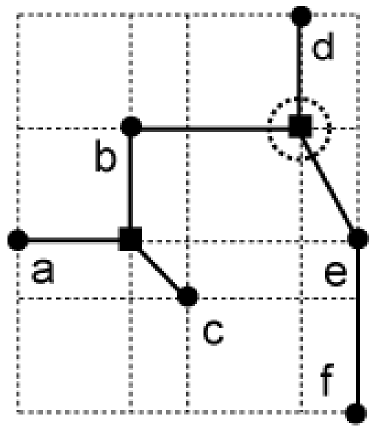
(f) WL=19

Second 1-Steiner Point Insertion

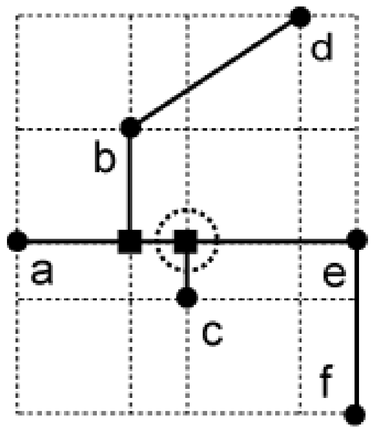
- Need to break tie again
  - Note that (a) and (b) do not contain any more 1-Steiner point: so we choose (c)



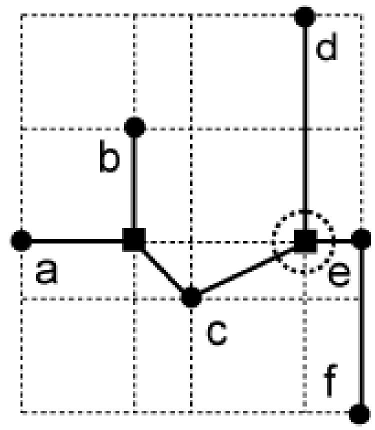
before insertion



(a) WL=17



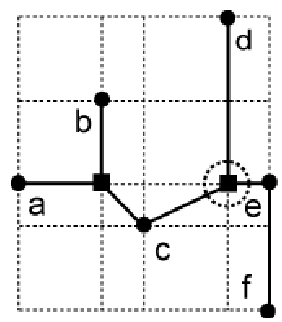
(b) WL=17



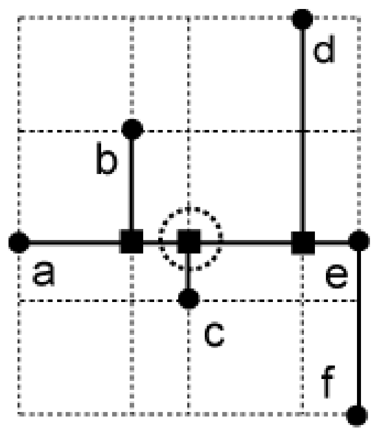
(c) WL=17

Third 1-Steiner Point Insertion

- Tree completed: all edges are rectilinearized
  - Overall wirelength reduction = 20 − 16 = 4



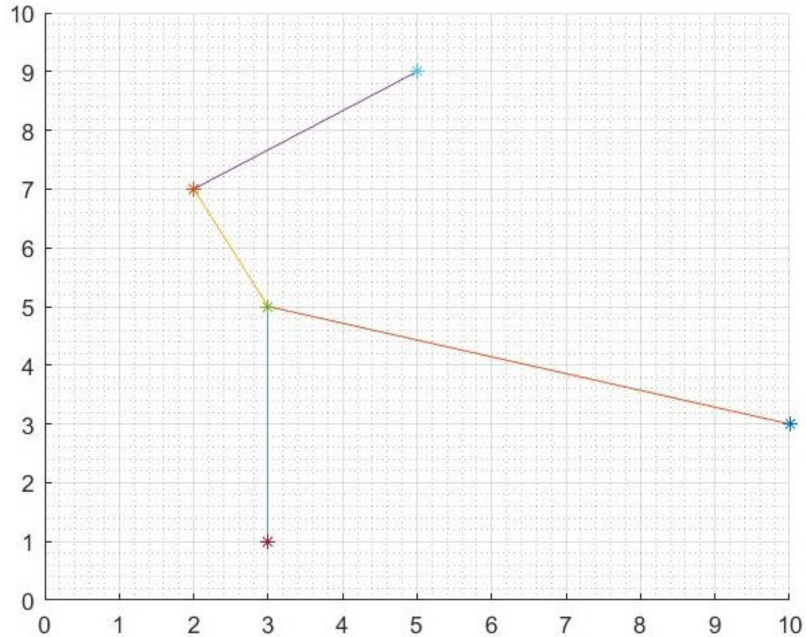
before insertion



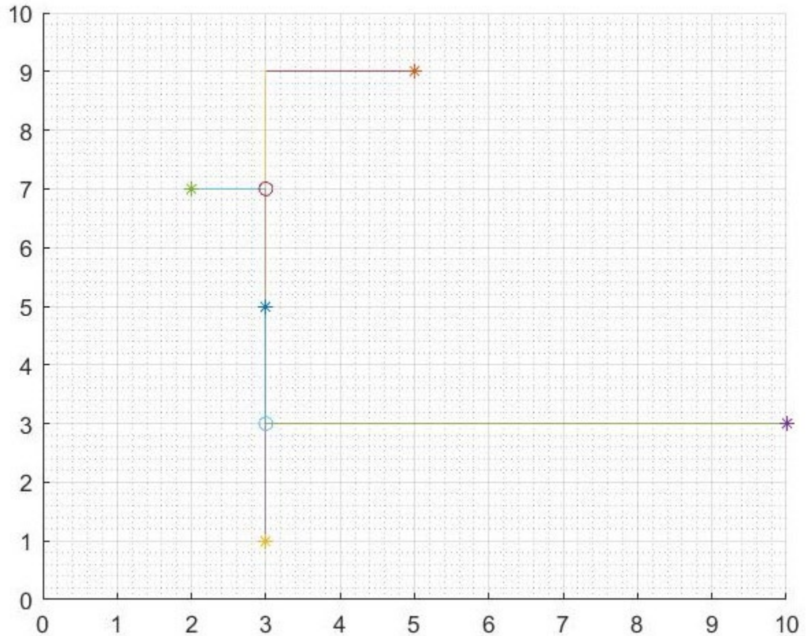
WL=16

Sample Kahng/Robins Routing (1/3)

- **5 points in 10x10 grid**
  - **2 Steiner points used**



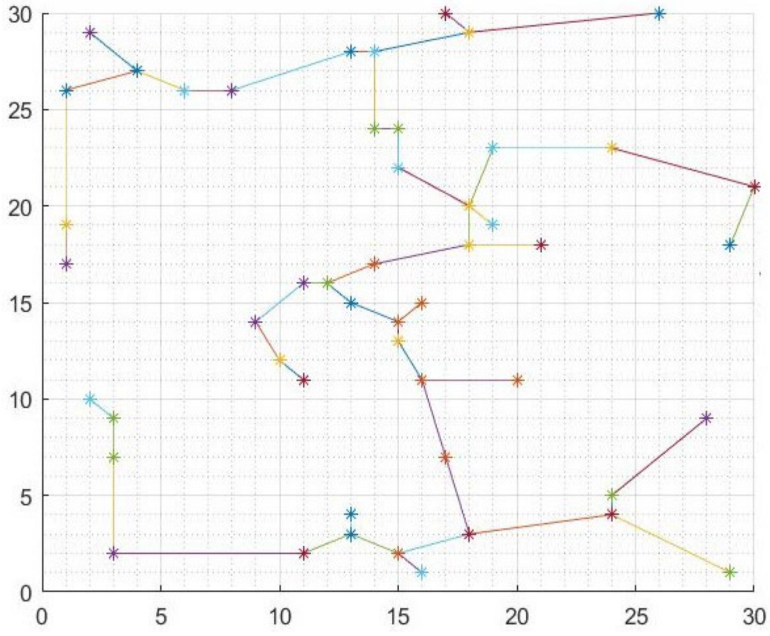
MST (WL = 21)



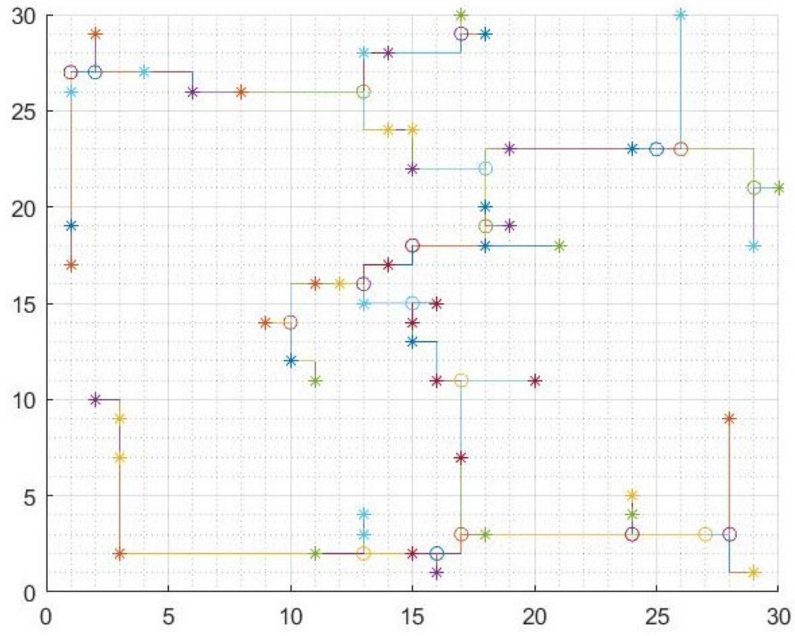
final tree (WL = 18)

Sample Kahng/Robins Routing (2/3)

- 50 points in 30x30 grid
  - 20 Steiner points used



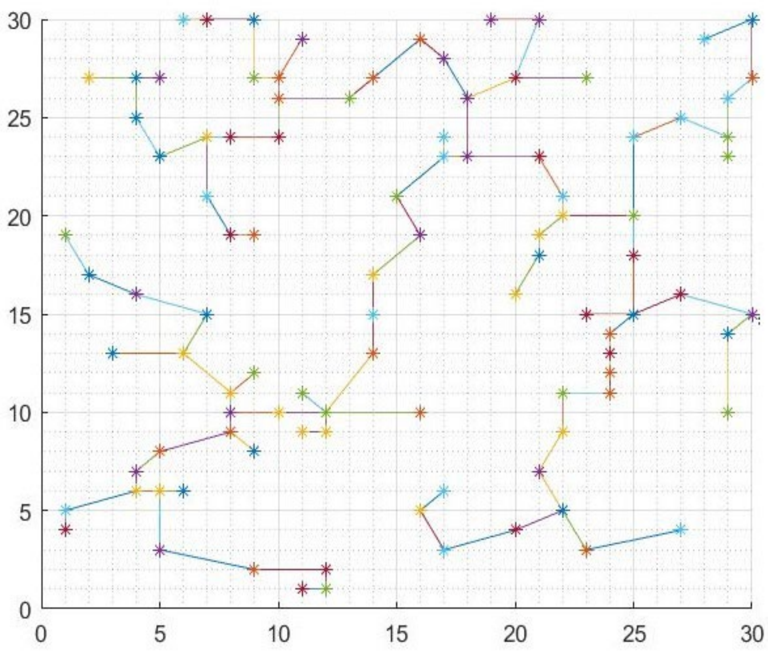
MST (WL = 183)



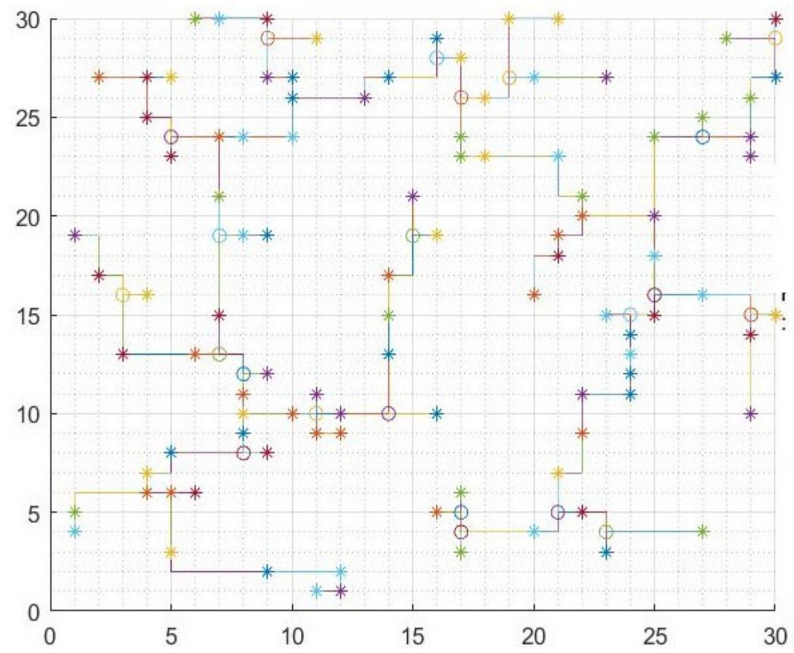
final tree (WL = 163)

Sample Kahng/Robins Routing (3/3)

- 100 points in 30x30 grid
  - 22 Steiner points used, it took 15ms to route



MST (WL = 242)



final tree (WL = 220)

Kahng/Robins Speedup Techniques

- Random variant
  - Instead of choosing the best gain Steiner point in each iteration, just pick the first one found.
  - Time spent on each step is less, but more Steiner points need to be added.

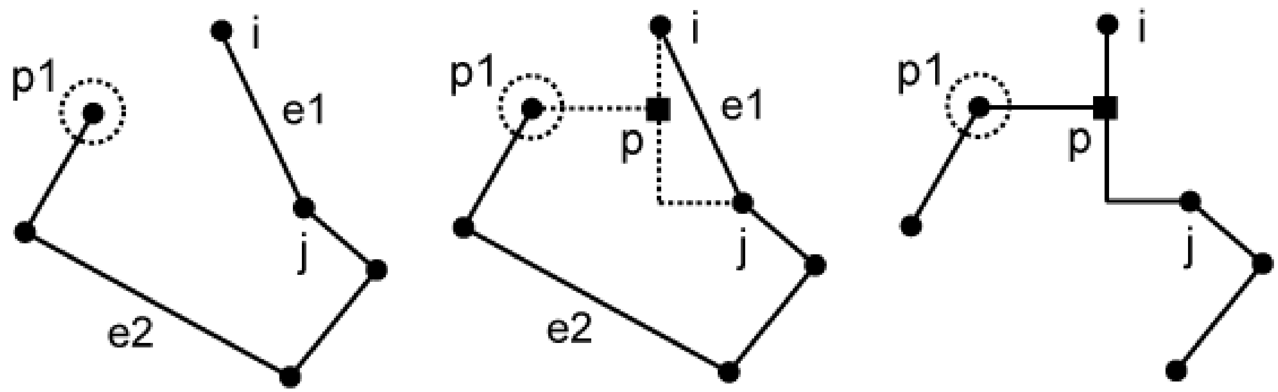
- Prune out bad candidates
  - After the first iteration, the Hanan grid points that gave no gain were removed.
  - This improved practical time complexity.

- Any other thoughts?

1-Steiner by Borah/Owens/Irwin

- **Interesting Observation**

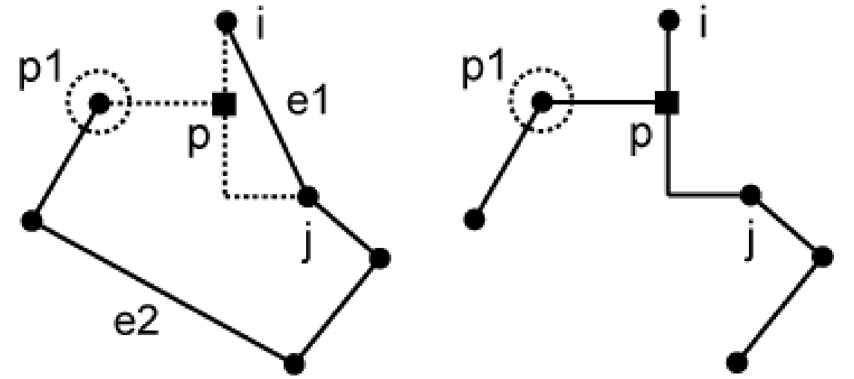
Our edge-based algorithm is based on connecting a node to the nearest point on the rectangular layout of an edge in the tree and removing the longest edge in the loop thus formed.



(a) (b) (c)

Gain Computation

- Things to do
  1) Add node p
  2) Remove edge e_1
  3) Remove edge e_2
  4) Add edge connecting p to p_1
  5) Add edge connecting p to p_2
  6) Add edge connecting p to p_3.



- Thus, the gain is

$$gain = length(e_2) - length(p, p_1)$$

Overall Algorithm

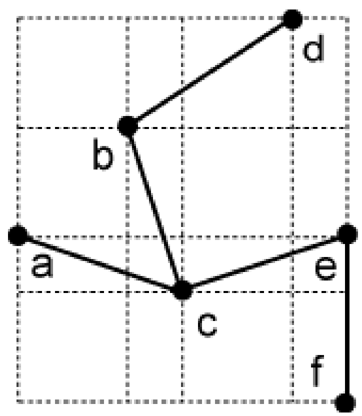
- Multi-pass Heuristic
  - Entire algorithm can be repeated

```
Algorithm Edge-based-Steiner()

Begin
   1.Compute the rectilinear minimum spanning tree of the set of nodes
   2.Compute all possible <node, edge> pairs that give positive gain
   3.Sort all the pairs in descending order of gain
   4.While (there are pairs with positive gain) do
            If (the two edges to be replaced exist in the tree) then
                Replace the pair of edges with three new edges and a new node.
            End-if
   End-while
End
```

1-Steiner Routing by Borah/Owens/Irwin

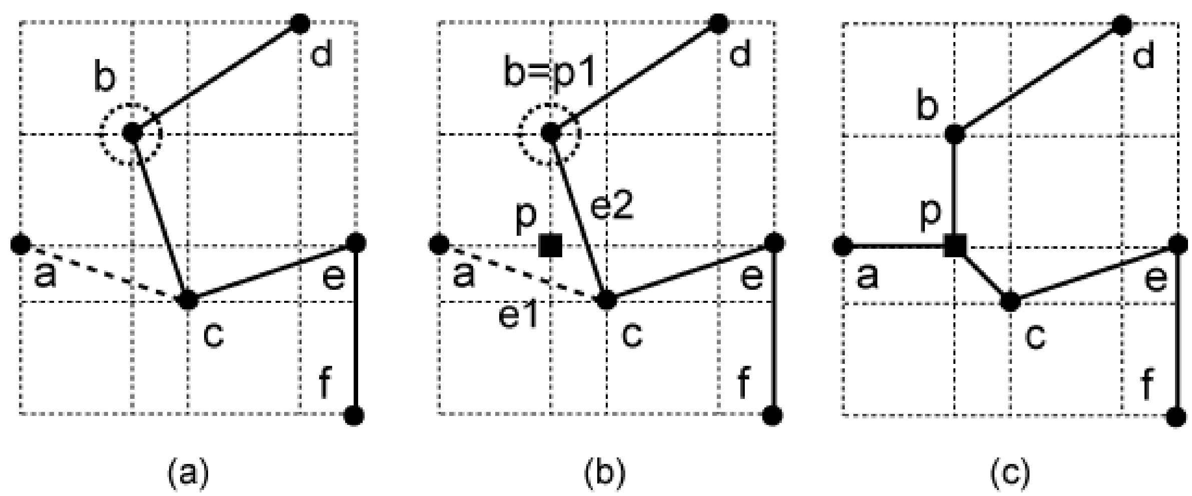
- Perform a single pass of Borah/Owens/Irwin
  - Initial MST has 5 edges with wirelength of 20
  - Need to compute the max-gain (node, edge) pair for each edge in this MST



Best Pair for (*a*,*c*)

We first let $p_1 = b$ and $e_1 = (a, c)$. Next, we compute the shortest Manhattan distance between p_1 and a "rectilinear layout" of e_1, which is 2 in this case. The node p is the nearest point on this rectilinear layout of e_1 to p_1. Next, we look for e_2, the longest edge on p_1-to-a path, which is $e_2 = (b, c)$. Thus,

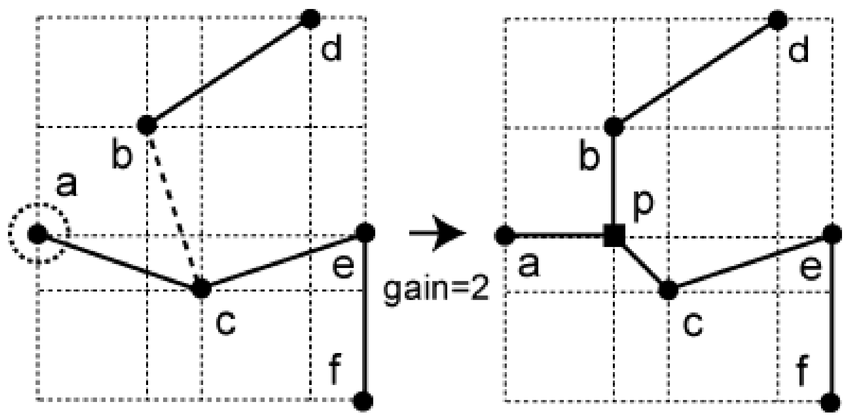
$$gain\{b, (a, c)\} = length(e_2) - length(p, p_1) = 4 - 2 = 2$$



Best Pair for (*b*,*c*)

- Three nodes can pair up with (*b*,*c*)

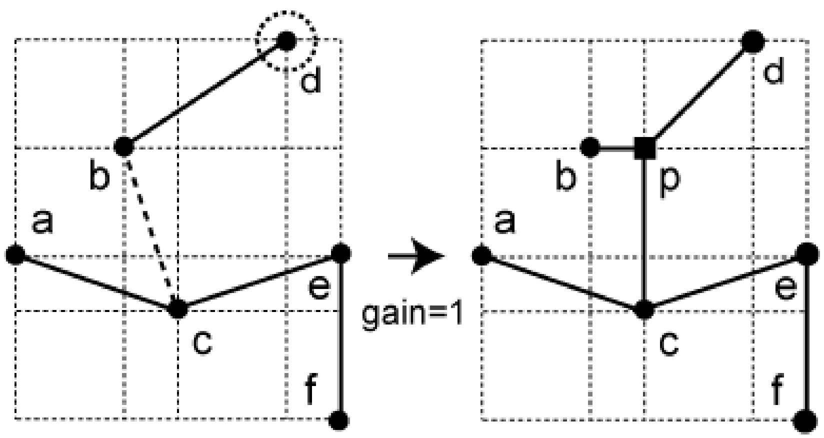
$$gain\{a,(b,c)\} = length(a,c) - length(p,a) = 4 - 2 = 2$$
$$gain\{d,(b,c)\} = length(b,d) - length(p,d) = 5 - 4 = 1$$
$$gain\{e,(b,c)\} = length(c,e) - length(p,e) = 4 - 3 = 1$$



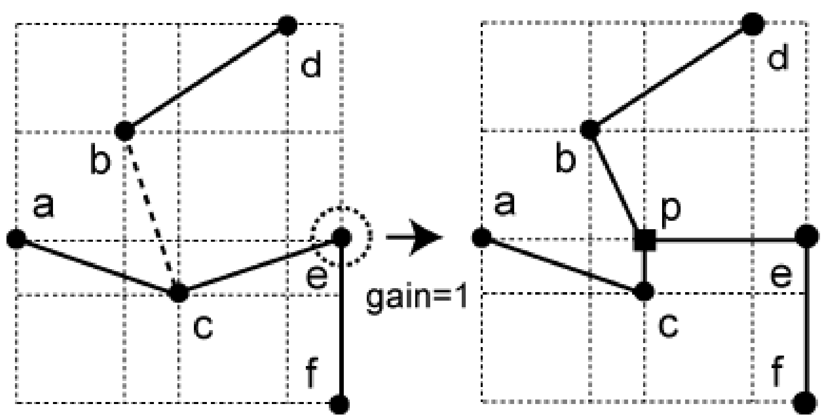
$l(a,c) - l(p,a) = 4 - 2$

Best Pair for (*b*,*c*) (cont)

- All three pairs have the same gain
  - Break ties randomly



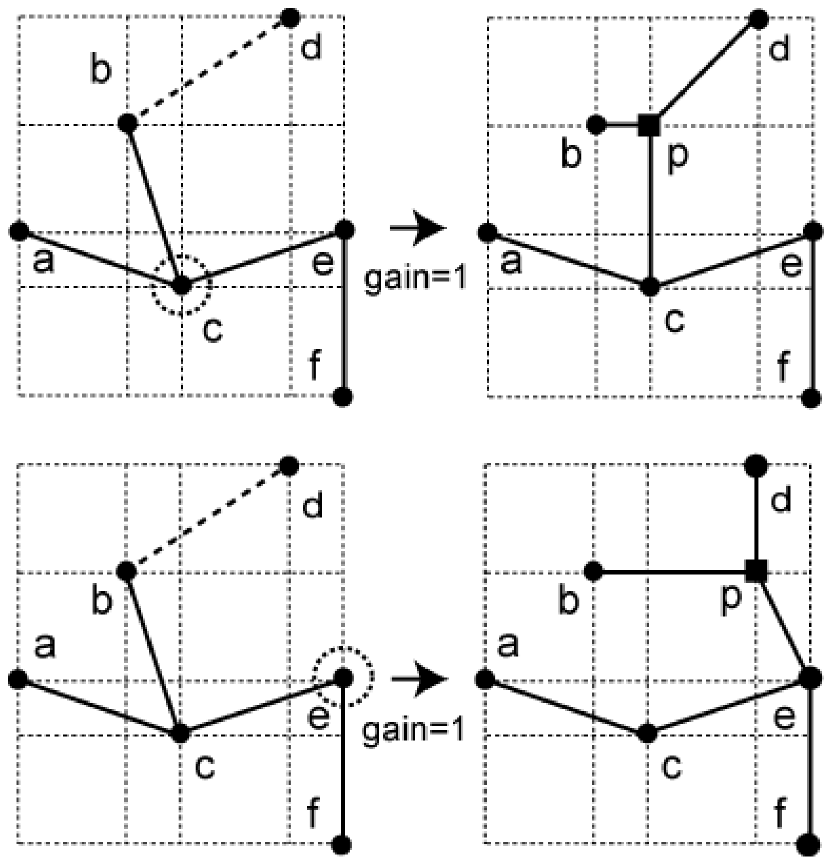
$l(b,d) - l(p,d) = 5 - 4$



$l(c,e) - l(p,e) = 4 - 3$

Best Pair for (b,d)

- Two nodes can pair up with (b,d)
  - both pairs have the same gain

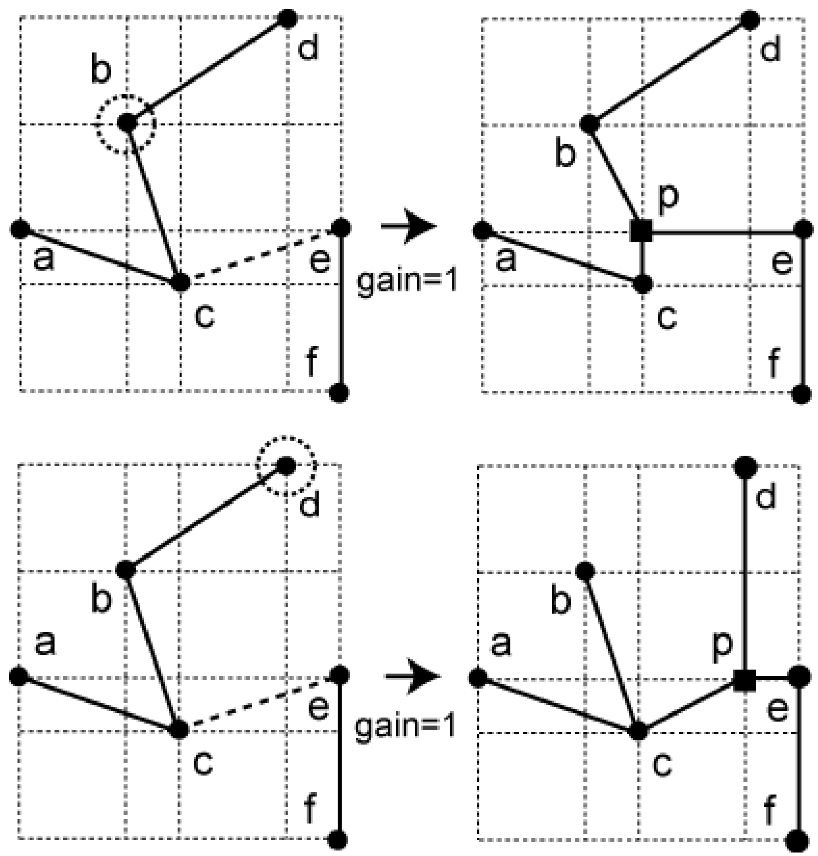


$l(b,c) - l(p,c) = 4 - 3$

$l(b,c) - l(p,e) = 4 - 3$

Best Pair for (c,e)

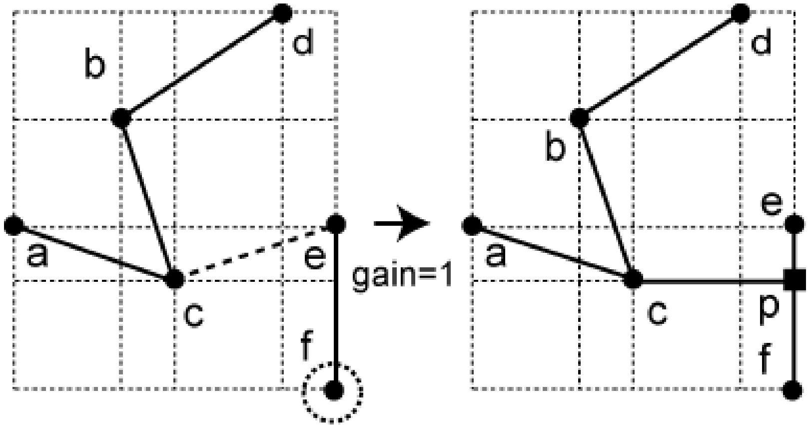
- Three nodes can pair up with (c,e)



$l(b,c) - l(p,b) = 4 - 3$

$l(b,d) - l(p,d) = 5 - 4$

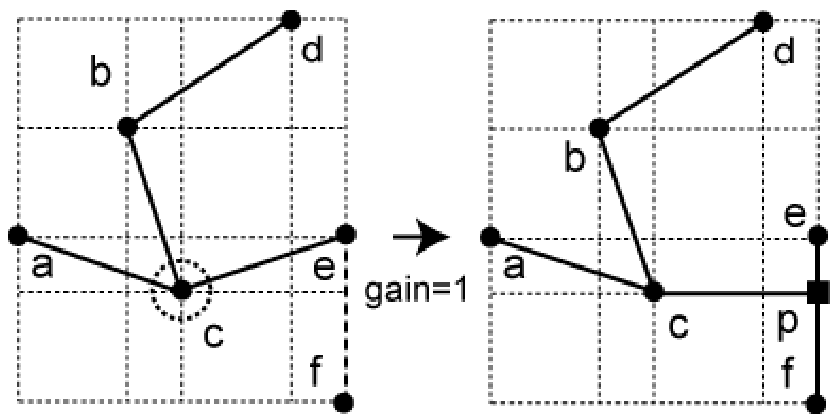
Best Pair for (*c*,*e*) (cont)



$l(e,f) - l(p,f) = 3 - 2$

Best Pair for (*e*,*f*)

- Can merge with *c* only

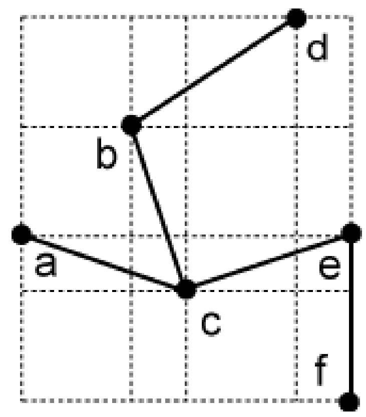


$l(c,e) - l(p,c) = 4 - 3$

Summary

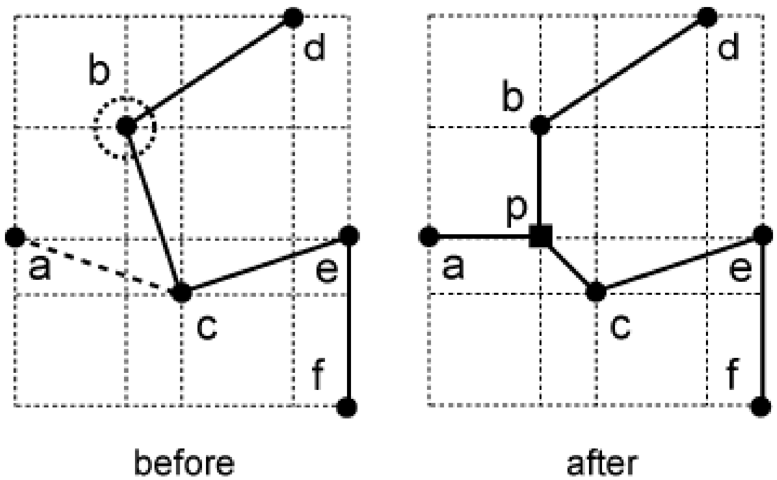
- Max-gain pair table
  - Sort based on gain value

pair	gain	e_1	e_2
$\{b, (a, c)\}$	2	(a, c)	(b, c)
$\{a, (b, c)\}$	2	(b, c)	(a, c)
$\{c, (b, d)\}$	1	(b, d)	(b, c)
$\{b, (c, e)\}$	1	(c, e)	(b, c)
$\{c, (e, f)\}$	1	(e, f)	(c, e)



First 1-Steiner Point Insertion

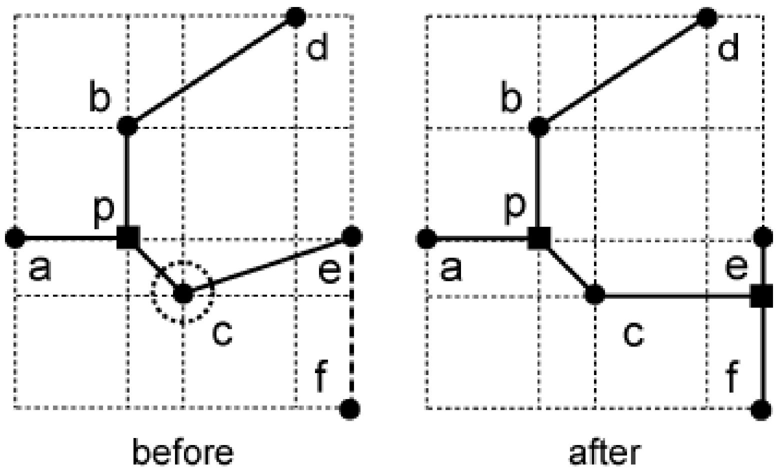
- Choose {*b*, (*a*,*c*)} (max-gain pair)
  - Mark $e_1 = (a,c)$, $e_2 = (b,c)$
  - Skip {*a*, (*b*,*c*)}, {*c*, (*b*,*d*)}, {*b*, (*c*,*e*)} since their e_1/e_2 are already marked
  - Wirelength reduces from 20 to 18



pair	gain	e_1	e_2
$\{b, (a, c)\}$	2	(a, c)	(b, c)
$\{a, (b, c)\}$	2	(b, c)	(a, c)
$\{c, (b, d)\}$	1	(b, d)	(b, c)
$\{b, (c, e)\}$	1	(c, e)	(b, c)
$\{c, (e, f)\}$	1	(e, f)	(c, e)

Second 1-Steiner Point Insertion

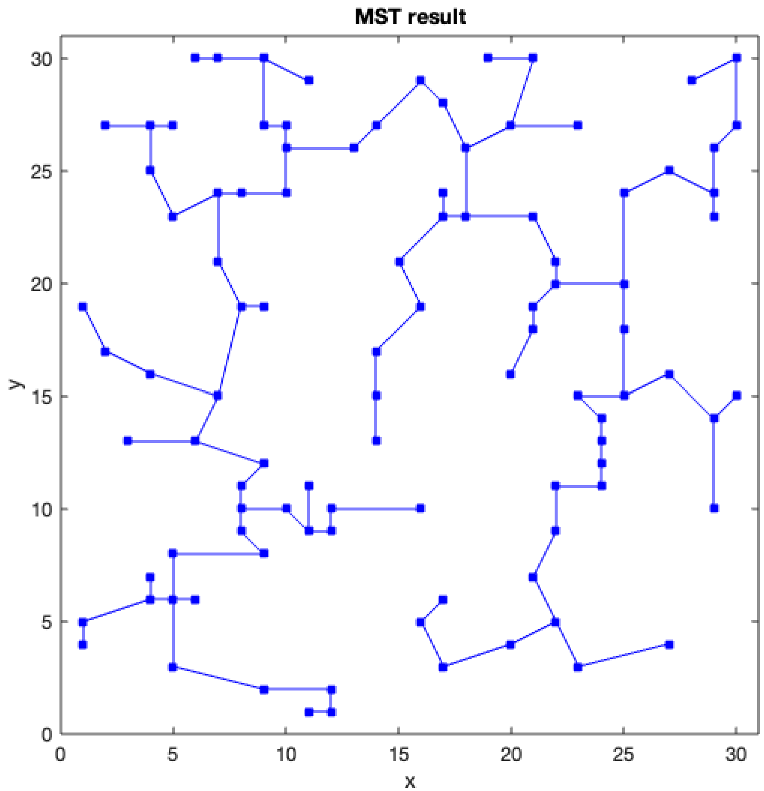
- Choose {c, (e,f)} (last one remaining)
  - Wirelength reduces from 18 to 17



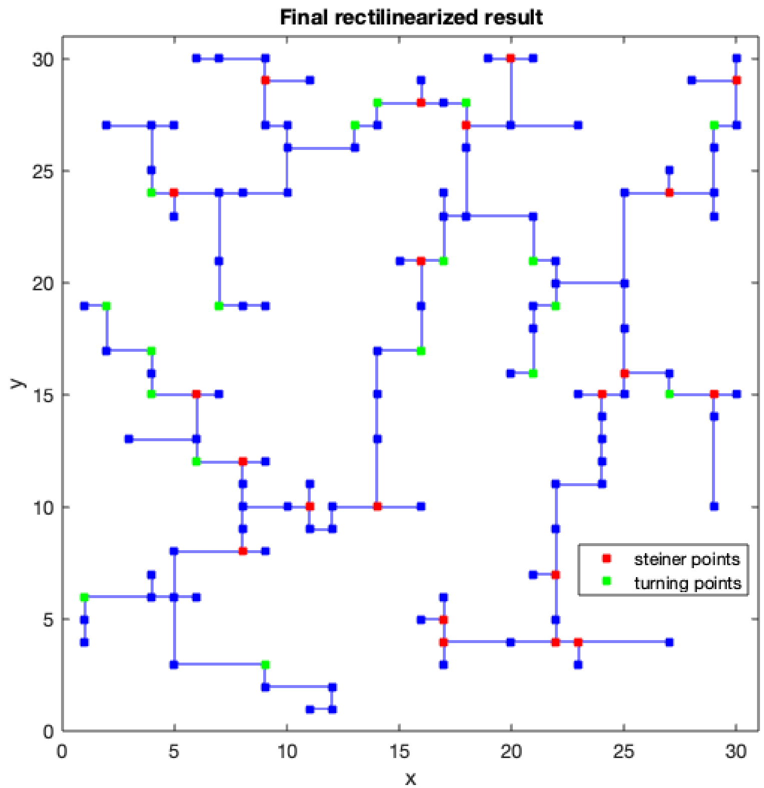
pair	gain	e_1	e_2
$\{b, (a, c)\}$	2	(a, c)	(b, c)
$\{a, (b, c)\}$	2	(b, c)	(a, c)
$\{c, (b, d)\}$	1	(b, d)	(b, c)
$\{b, (c, e)\}$	1	(c, e)	(b, c)
$\{c, (e, f)\}$	1	(e, f)	(c, e)

Sample Borah Routing

- **100 points in 30x30 grid**
  - **22 Steiner points used, it took 59ms to route**



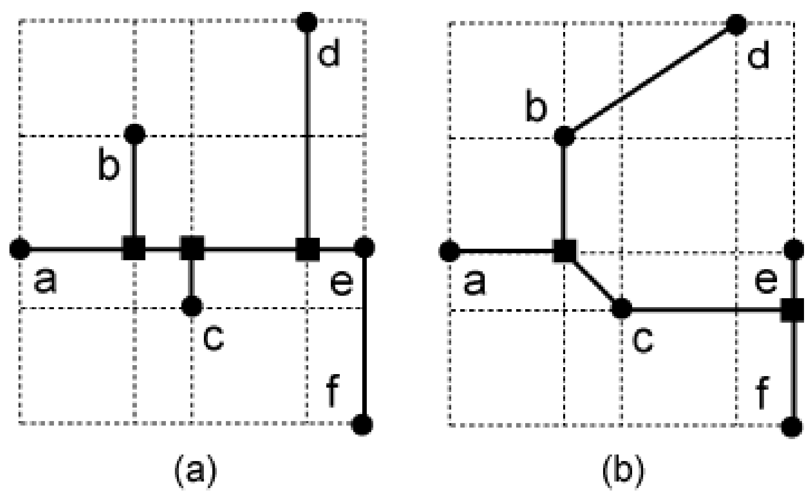
MST (WL = 242)



final tree (WL = 219)

Comparison

- Kahng/Robins vs Borah/Owens/Irwin
  - Kahng/Robins tends to give better results
  - Borah/Owens/Irwin runs much faster: $O(n^4 \log n)$ vs $O(n^2)$



Bounded Radius Routing

- Why Radius?
  - Longest source-sink path length among all sinks
  - Smaller path resistance: better performance
- Both Radius and Cost?
  - Cost = wirelength
  - Radius (= R) and wirelength (= C) are both important for RC-delay reduction
- Bounded PRIM vs Bounded Radius/Cost
  - J. Cong, A. B. Kahng, G. Robins, M. Sarrafzadeh, and C. K. Wong, "Provably good performance-driven global routing", TCAD, 1992.

Radius vs Wirelength

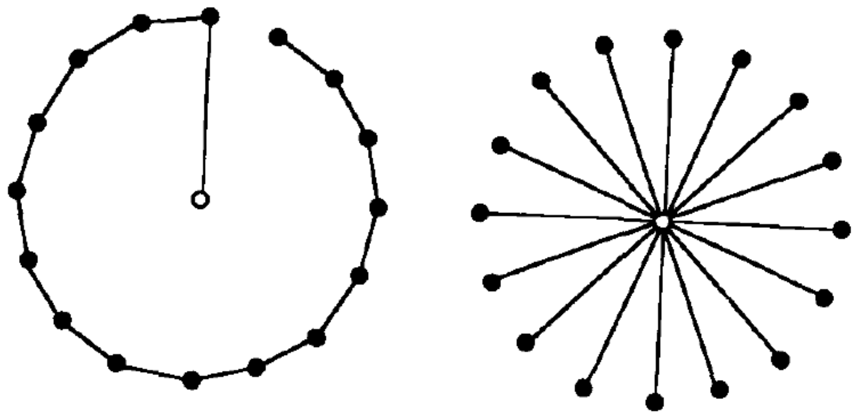


Fig. 1. An example where the cost of a shortest path tree (*right*) is $\Omega(|N|)$ times larger than the cost of a minimum spanning tree (*left*).

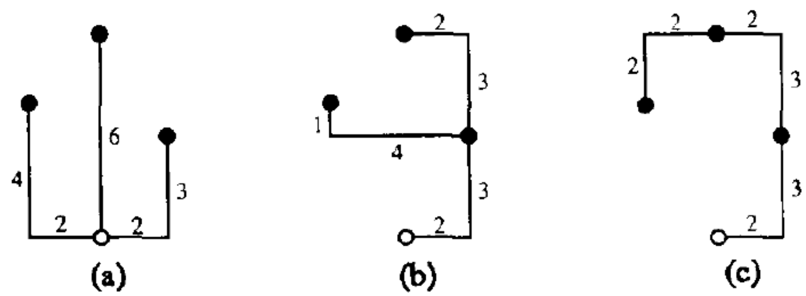
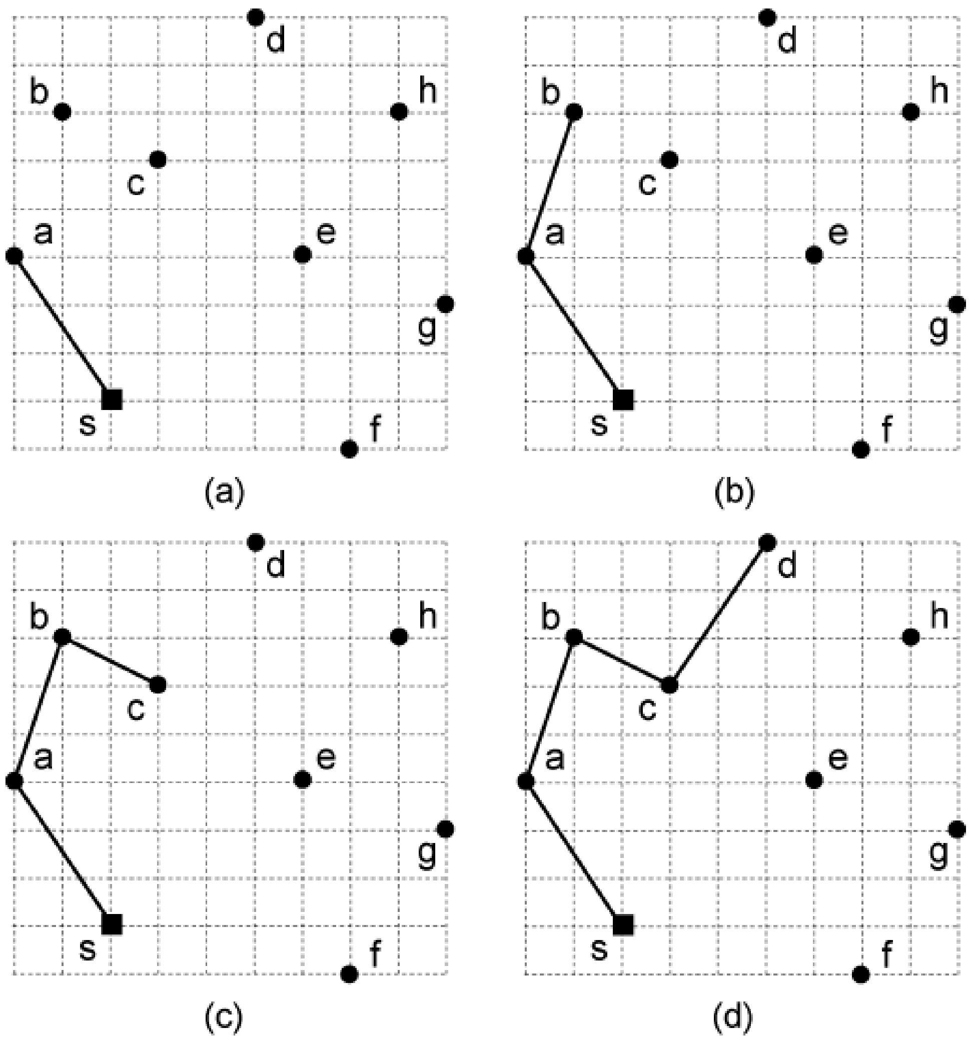


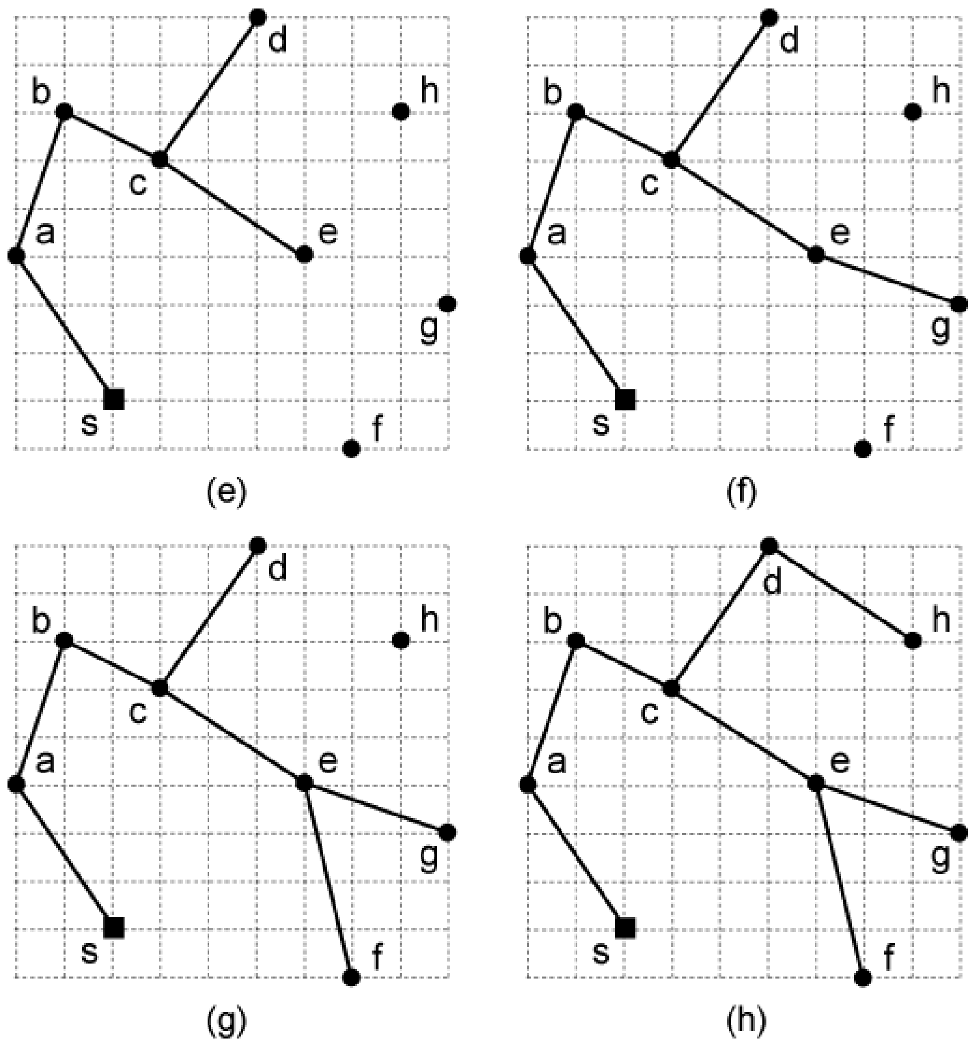
Fig. 2. An example in the Manhattan plane of how increasing the value of ϵ may result in decreased tree cost, but increased radius $r(T)$: (a) $\epsilon = 0$, cost$(T) = 17$, $n(T) = 6$; (b) $\epsilon = 1$, cost$(T) = 15$, $n(T) = 10$; (c) $\epsilon = \infty$, cost$(T) = 14$, $r(T) = 14$.

BPRIM Under $\varepsilon = \infty$

Radius bound = ∞
= regular PRIM



BPRIM Under $\varepsilon = \infty$ (cont)



Bounded PRIM Algorithm

- **Variation of PRIM's MST algorithm**

$T = (V', E') = (\{s\}, \emptyset)$
while $|V'| < |N|$
  Select two terminals $x \in V'$ and $y \in N - V'$ minimizing $dist(x, y)$
  if $dist_T(s, x) + dist(x, y) \leq (1 + \epsilon) \cdot R$ **then** $x' = x$
  else find the first terminal x' along the path in T from x to s
    such that $dist_T(s, x') + dist(x', y) \leq R$
  $V' = V' \cup \{x'\}$
  $E' = E' \cup \{(x', y)\}$

Fig. 4. Algorithm BPRIM: computing a bounded-radius spanning tree, T, for a given set of terminals, N, with source, $s \in N$, and radius, R, using parameter ϵ.

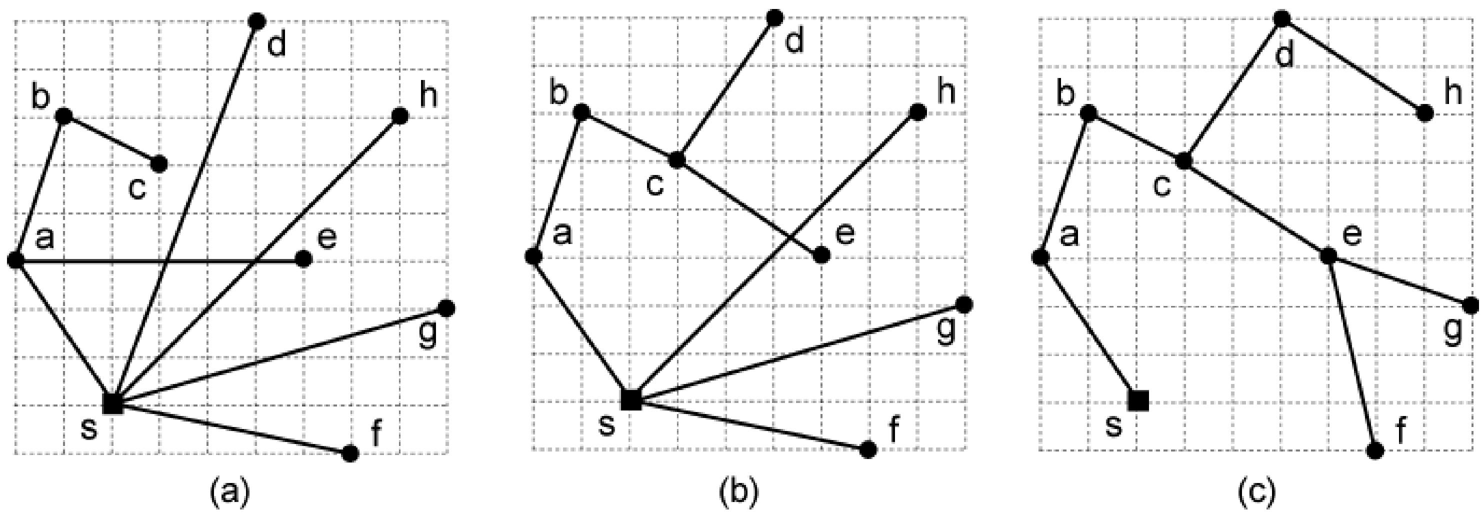
Why Tighter Radius?

- **BPRIM uses tighter radius bound during backtracing**
  - **R instead of (1+e)R**

Note that in backtracing we could choose x' such that $dist_T(s, x') + dist(x', y) \leq (1 + \epsilon) \cdot R$. However, our choice of appropriate edges leads to fewer backtracing operations, while guaranteeing that backtracing is still always possible. In other words, we intentionally introduce some "slack" at y so that terminals within an ϵR neighborhood of y will not cause additional backtracing. Limiting the amount of backtracing in this way will keep the cost of the resulting tree close to that of the minimum spanning tree.

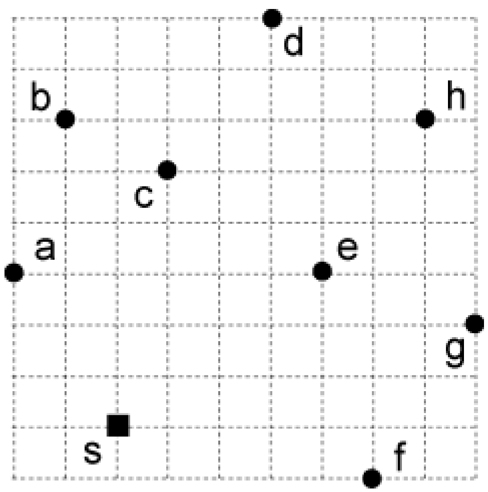
Bounded PRIM Algorithm

- **Comparison (e = 0, 0.5, infinity)**
  - **Radius bound/value increase**
  - **Wirelength decreases**

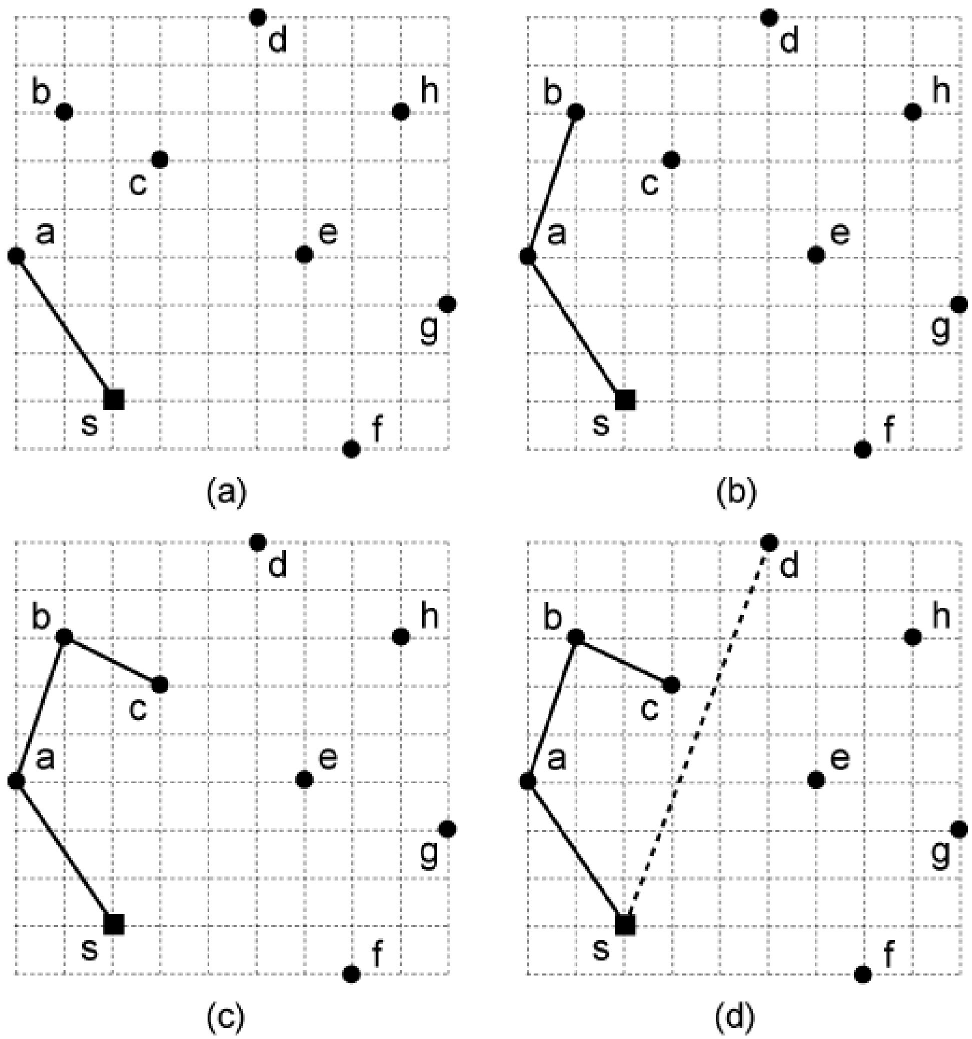


Bounded Radius Routing

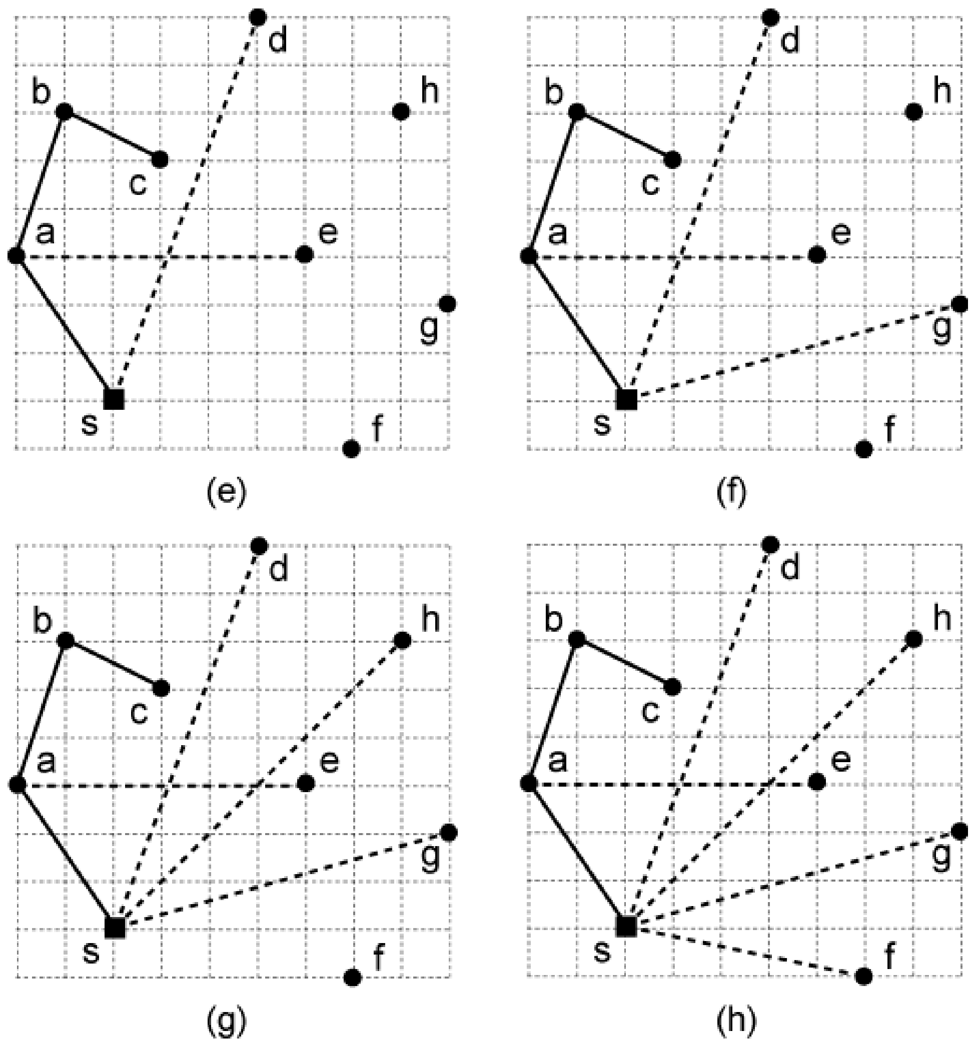
- Perform bounded PRIM algorithm
  - Under $\varepsilon = 0$, $\varepsilon = 0.5$, and $\varepsilon = \infty$
  - Compare radius and wirelength
  - Radius = 12 for this net



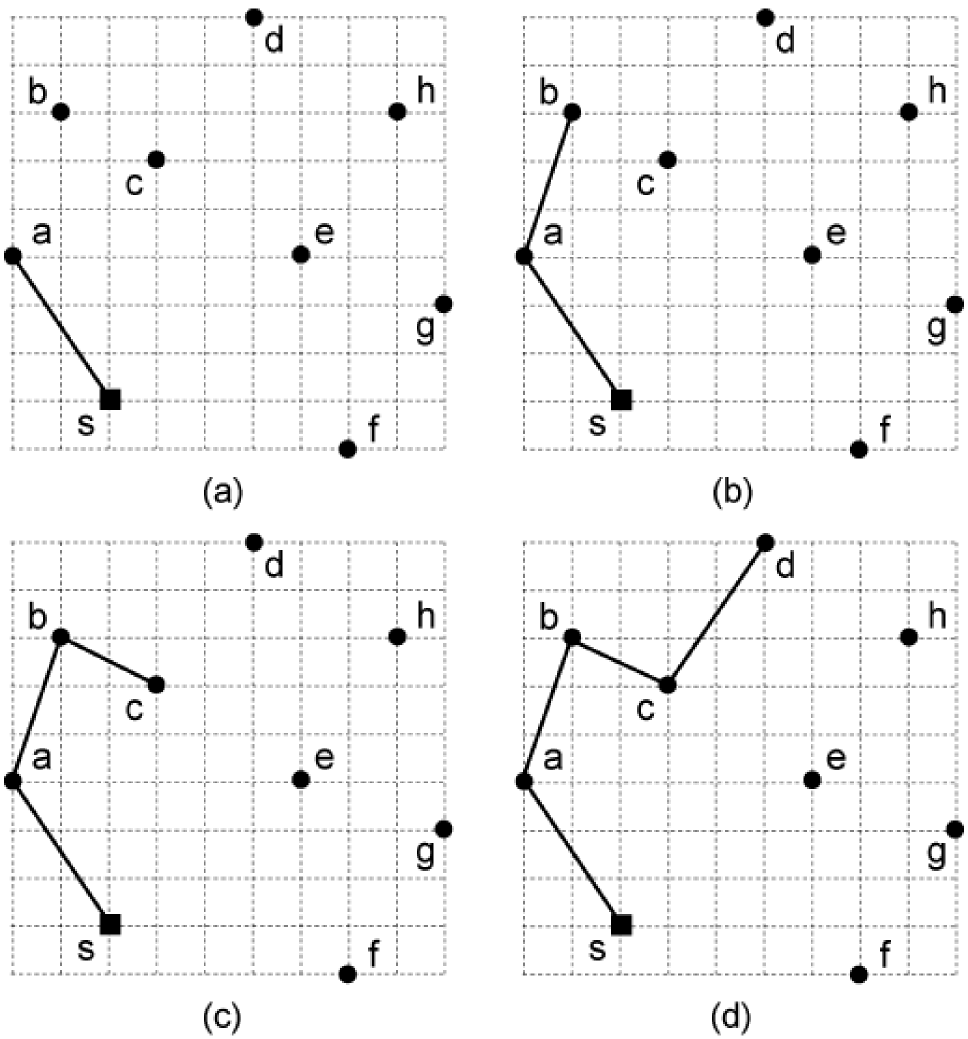
BPRIM Under $\varepsilon = 0$ (cont)



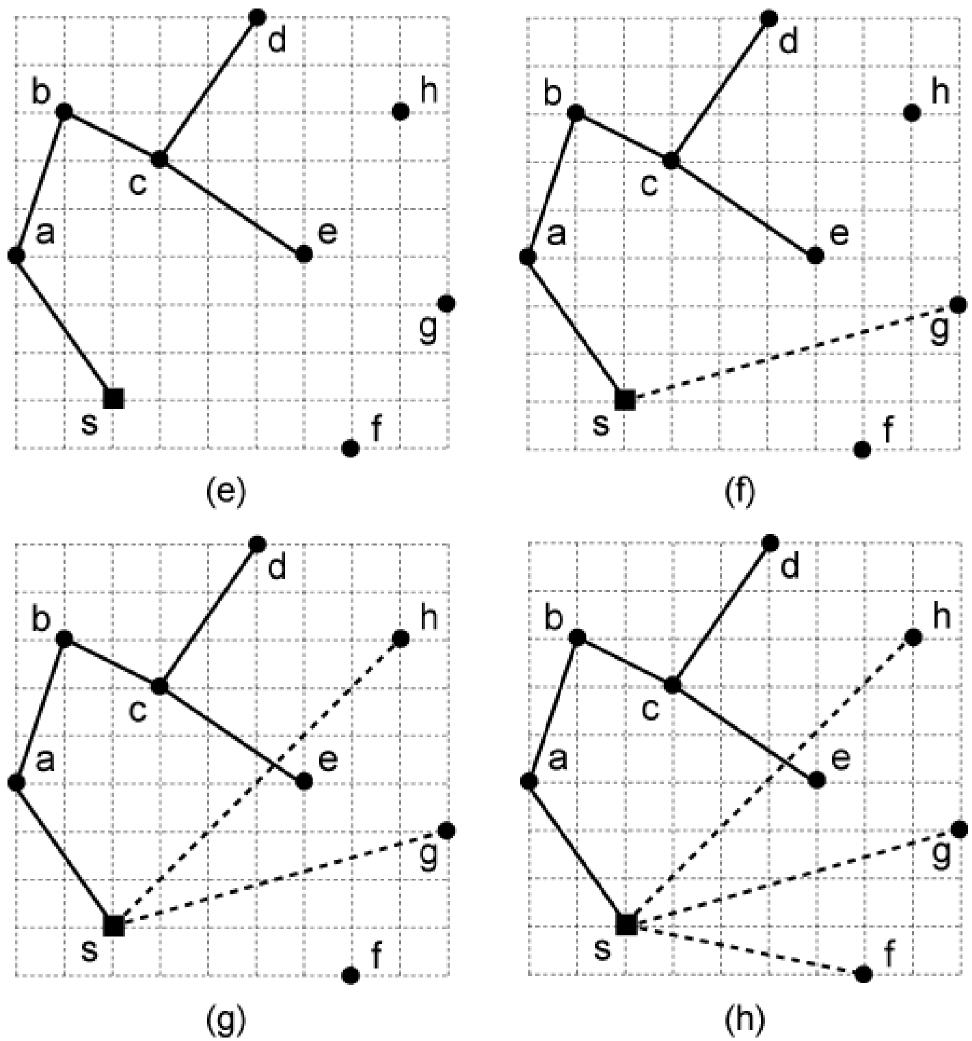
BPRIM Under $\varepsilon = 0$ (cont)



BPRIM Under $\varepsilon = 0.5$ (cont)



BPRIM Under $\varepsilon = 0.5$ (cont)



Comparison

- As the bound increases (12 → 18 → ∞)
  - Radius value increases (12 →17 → 22)
  - Wirelength decreases (56 → 49 → 36)