I. Floorplanning with Fixed Modules

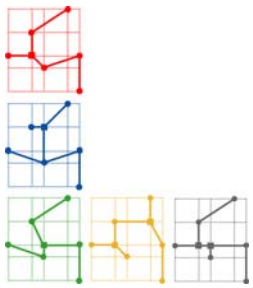
- Fixed modules only, no rotation allowed
  - m_1 (4,5), m_2 (3,7), m_3 (6,4), m_4 (7,7)

First, we obtain the list of continuous and integer variables as follows:

- 8 continuous variables: the coordinate variables (x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4)
- 12 integer variables: the all-pair relative position variables (x_{12}, x_{13}, x_{14}, x_{23}, x_{24}, x_{34}, y_{12}, y_{13}, y_{14}, y_{23}, y_{24}, y_{34}).

The upper bound of chip boundary is computed as follows:

$$W = \sum w_i = 4 + 3 + 6 + 7 = 20$$
$$H = \sum h_i = 5 + 7 + 4 + 7 = 23$$

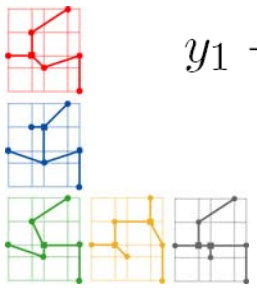


ILP Formulation

Minimize y^*
Subject to
non-overlap constraints:

$$x_1 + w_1 \leq x_2 + 20(x_{12} + y_{12})$$
$$x_1 - w_2 \geq x_2 - 20(1 - x_{12} + y_{12})$$
$$y_1 + h_1 \leq y_2 + 23(1 + x_{12} - y_{12})$$
$$y_1 - h_2 \geq y_2 - 23(2 - x_{12} - y_{12})$$

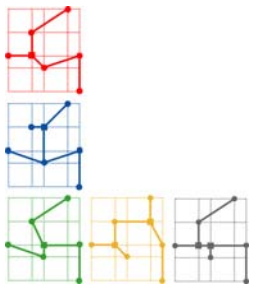
$$x_1 + w_1 \leq x_3 + 20(x_{13} + y_{13})$$
$$x_1 - w_3 \geq x_3 - 20(1 - x_{13} + y_{13})$$
$$y_1 + h_1 \leq y_3 + 23(1 + x_{13} - y_{13})$$
$$y_1 - h_3 \geq y_3 - 23(2 - x_{13} - y_{13})$$



Non-Overlapping Constraints (cont)

$$x_1 + w_1 \leq x_4 + 20(x_{14} + y_{14}) \qquad x_2 + w_2 \leq x_4 + 20(x_{24} + y_{24})$$
$$x_1 - w_4 \geq x_4 - 20(1 - x_{14} + y_{14}) \qquad x_2 - w_4 \geq x_4 - 20(1 - x_{24} + y_{24})$$
$$y_1 + h_1 \leq y_4 + 23(1 + x_{14} - y_{14}) \qquad y_2 + h_2 \leq y_4 + 23(1 + x_{24} - y_{24})$$
$$y_1 - h_4 \geq y_4 - 23(2 - x_{14} - y_{14}) \qquad y_2 - h_4 \geq y_4 - 23(2 - x_{24} - y_{24})$$

$$x_2 + w_2 \leq x_3 + 20(x_{23} + y_{23}) \qquad x_3 + w_3 \leq x_4 + 20(x_{34} + y_{34})$$
$$x_2 - w_3 \geq x_3 - 20(1 - x_{23} + y_{23}) \qquad x_3 - w_4 \geq x_4 - 20(1 - x_{34} + y_{34})$$
$$y_2 + h_2 \leq y_3 + 23(1 + x_{23} - y_{23}) \qquad y_3 + h_3 \leq y_4 + 23(1 + x_{34} - y_{34})$$
$$y_2 - h_3 \geq y_3 - 23(2 - x_{23} - y_{23}) \qquad y_3 - h_4 \geq y_4 - 23(2 - x_{34} - y_{34})$$



Additional Constraints

variable type constraints:

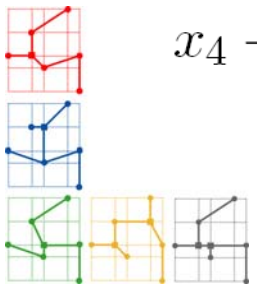
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$$
$$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0,\ y_4 \geq 0$$
$$x_{12},\ x_{13},\ x_{14},\ x_{23},\ x_{24},\ x_{34} \in \{0, 1\}$$
$$y_{12},\ y_{13},\ y_{14},\ y_{23},\ y_{24},\ y_{34} \in \{0, 1\}$$

chip width constraints:

$$x_1 + w_1 \leq y^*$$
$$x_2 + w_2 \leq y^*$$
$$x_3 + w_3 \leq y^*$$
$$x_4 + w_4 \leq y^*$$

chip height constraints:

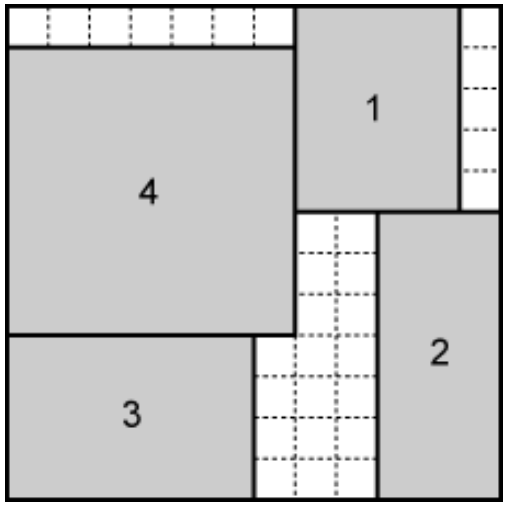
$$y_1 + h_1 \leq y^*$$
$$y_2 + h_2 \leq y^*$$
$$y_3 + h_3 \leq y^*$$
$$y_4 + h_4 \leq y^*$$

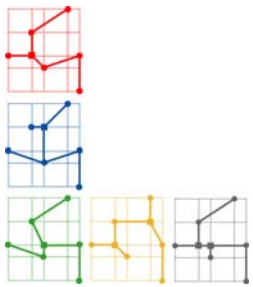


Solutions

- Using GLPK we get the following solutions:

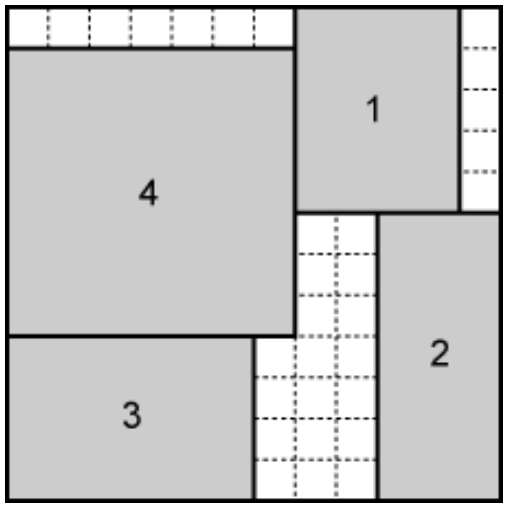
$$y^* = 12$$
$$(x_1, y_1) = (7, 7), (x_2, y_2) = (9, 0), (x_3, y_3) = (0, 0), (x_4, y_4) = (0, 4)$$
$$(x_{12}, y_{12}) = (1, 1) : (1 \text{ is above } 2)$$
$$(x_{13}, y_{13}) = (1, 1) : (1 \text{ is above } 3)$$
$$(x_{14}, y_{14}) = (1, 0) : (1 \text{ is to the right of } 4)$$
$$(x_{23}, y_{23}) = (1, 0) : (2 \text{ is to the right of } 3)$$
$$(x_{24}, y_{24}) = (1, 0) : (2 \text{ is to the right of } 4)$$
$$(x_{34}, y_{34}) = (0, 1) : (3 \text{ is below } 4)$$

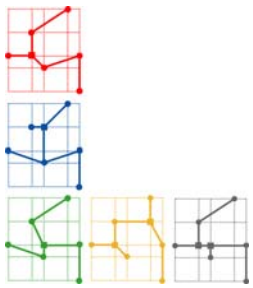




Final Floorplan

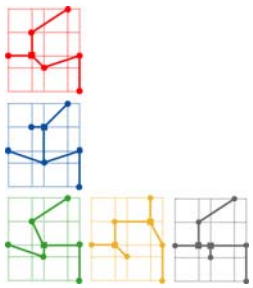
- Why the non-optimality?
  - Due to linear approximation of area objective (= y^*)
  - Chip width/height constraints also affected
  - In fact, our ILP solution ($y^* = 12$) is optimal under these conditions.





II. Floorplanning with Rotation

- Fixed modules, rotation allowed
  - Fixed modules: m_1 (4,5), m_2 (3,7), m_3 (6,4), m_4 (7,7)
  - Need 4 more binary variables for rotation: z_1, z_2, z_3, z_4
  - We use $M = \max\{W,H\} = 23$

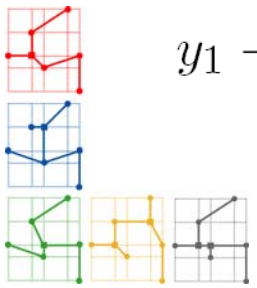


ILP Formulation

Minimize y^*
Subject to
non-overlap constraints:

$$x_1 + z_1 h_1 + (1 - z_1) w_1 \leq x_2 + 23(x_{12} + y_{12})$$
$$x_1 - z_2 h_2 - (1 - z_2) w_2 \geq x_2 - 23(1 - x_{12} + y_{12})$$
$$y_1 + z_1 w_1 + (1 - z_1) h_1 \leq y_2 + 23(1 + x_{12} - y_{12})$$
$$y_1 - z_2 w_2 - (1 - z_2) h_2 \geq y_2 - 23(2 - x_{12} - y_{12})$$

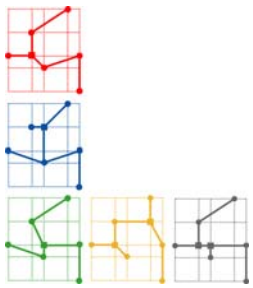
$$x_1 + z_1 h_1 + (1 - z_1) w_1 \leq x_3 + 23(x_{13} + y_{13})$$
$$x_1 - z_3 h_3 - (1 - z_3) w_3 \geq x_3 - 23(1 - x_{13} + y_{13})$$
$$y_1 + z_1 w_1 + (1 - z_1) h_1 \leq y_3 + 23(1 + x_{13} - y_{13})$$
$$y_1 - z_3 w_3 - (1 - z_3) h_3 \geq y_3 - 23(2 - x_{13} - y_{13})$$



Non-Overlapping Constraints (cont)

$$x_1 + z_1 h_1 + (1 - z_1) w_1 \leq x_4 + 23(x_{14} + y_{14})$$
$$x_1 - z_4 h_4 - (1 - z_4) w_4 \geq x_4 - 23(1 - x_{14} + y_{14})$$
$$y_1 + z_1 w_1 + (1 - z_1) h_1 \leq y_4 + 23(1 + x_{14} - y_{14})$$
$$y_1 - z_4 w_4 - (1 - z_4) h_4 \geq y_4 - 23(2 - x_{14} - y_{14})$$

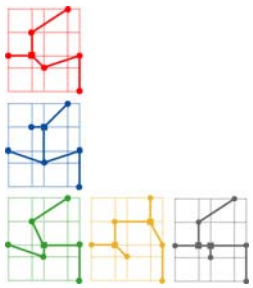
$$x_2 + z_2 h_2 + (1 - z_2) w_2 \leq x_3 + 23(x_{23} + y_{23})$$
$$x_2 - z_3 h_3 - (1 - z_3) w_3 \geq x_3 - 23(1 - x_{23} + y_{23})$$
$$y_2 + z_2 w_2 + (1 - z_2) h_2 \leq y_3 + 23(1 + x_{23} - y_{23})$$
$$y_2 - z_3 w_3 - (1 - z_3) h_3 \geq y_3 - 23(2 - x_{23} - y_{23})$$



Non-Overlapping Constraints (cont)

$$x_2 + z_2 h_2 + (1 - z_2) w_2 \leq x_4 + 23(x_{24} + y_{24})$$
$$x_2 - z_4 h_4 - (1 - z_4) w_4 \geq x_4 - 23(1 - x_{24} + y_{24})$$
$$y_2 + z_2 w_2 + (1 - z_2) h_2 \leq y_4 + 23(1 + x_{24} - y_{24})$$
$$y_2 - z_4 w_4 - (1 - z_4) h_4 \geq y_4 - 23(2 - x_{24} - y_{24})$$

$$x_3 + z_3 h_3 + (1 - z_3) w_3 \leq x_4 + 23(x_{34} + y_{34})$$
$$x_3 - z_4 h_4 - (1 - z_4) w_4 \geq x_4 - 23(1 - x_{34} + y_{34})$$
$$y_3 + z_3 w_3 + (1 - z_3) h_3 \leq y_4 + 23(1 + x_{34} - y_{34})$$
$$y_3 - z_4 w_4 - (1 - z_4) h_4 \geq y_4 - 23(2 - x_{34} - y_{34})$$



Additional Constraints

variable type constraints:

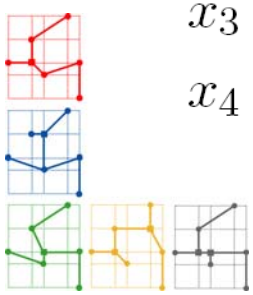
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$$
$$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0,\ y_4 \geq 0$$
$$x_{12},\ x_{13},\ x_{14},\ x_{23},\ x_{24},\ x_{34} \in \{0, 1\}$$
$$y_{12},\ y_{13},\ y_{14},\ y_{23},\ y_{24},\ y_{34} \in \{0, 1\}$$
$$z_1,\ z_2,\ z_3,\ z_4 \in \{0, 1\}$$

chip width constraints:

$$x_1 + (1 - z_1)w_1 + z_1 h_1 \leq y^*$$
$$x_2 + (1 - z_2)w_2 + z_2 h_2 \leq y^*$$
$$x_3 + (1 - z_3)w_3 + z_3 h_3 \leq y^*$$
$$x_4 + (1 - z_4)w_4 + z_4 h_4 \leq y^*$$

chip height constraints:

$$y_1 + (1 - z_1)h_1 + z_1 w_1 \leq y^*$$
$$y_2 + (1 - z_2)h_2 + z_2 w_2 \leq y^*$$
$$y_3 + (1 - z_3)h_3 + z_3 w_3 \leq y^*$$
$$y_4 + (1 - z_4)h_4 + z_4 w_4 \leq y^*$$



Solutions

- Using GLPK we get the following solutions:

$y^* = 11$

$(x_1, y_1) = (7, 6), (x_2, y_2) = (0, 0), (x_3, y_3) = (7, 0), (x_4, y_4) = (0, 3)$

$z_1 = 0, z_2 = 1, z_3 = 1, z_4 = 0$: (2 and 3 are rotated.)

$(x_{12}, y_{12}) = (1, 1) : (1 \text{ is above } 2)$

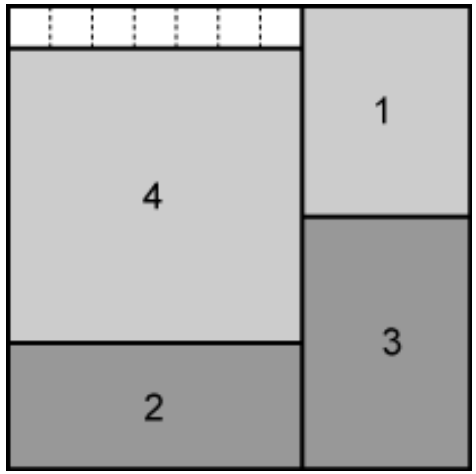
$(x_{13}, y_{13}) = (1, 1) : (1 \text{ is above } 3)$

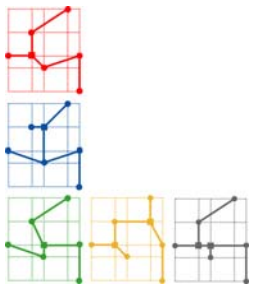
$(x_{14}, y_{14}) = (1, 0) : (1 \text{ is to the right of } 4)$

$(x_{23}, y_{23}) = (0, 0) : (2 \text{ is to the left of } 3)$

$(x_{24}, y_{24}) = (0, 1) : (2 \text{ is below } 4)$

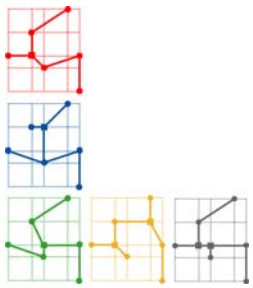
$(x_{34}, y_{34}) = (1, 0) : (3 \text{ is to the right of } 4)$





III. Floorplanning with Flexible Modules

- 2 Fixed modules:
  - m_1 (4,5), m_2 (3,7) (rotation allowed)
- 2 Flexible modules:
  - m_3: area = 24, aspect ratio [0.5, 2]
  - m_4: area = 49, aspect ratio [0.3, 2.5]
- 10 continuous variables: the coordinate variables (x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4), and the sizing variables (w_3, w_4) for the flexible modules.
- 14 integer variables: the all-pair relative position variables (x_{12}, x_{13}, x_{14}, x_{23}, x_{24}, x_{34}, y_{12}, y_{13}, y_{14}, y_{23}, y_{24}, y_{34}), and the rotation variables (z_1, z_2) for the fixed modules.



Linear Approximation

From the area and aspect ratio constraints, we get

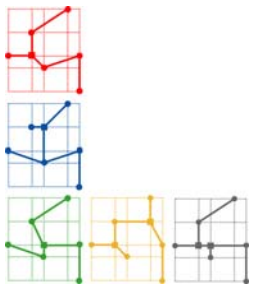
$$w_i \cdot h_i \geq S_i,\ l_i \leq \frac{w_i}{h_i} \leq u_i$$
$$w_{i,min} = \sqrt{S_i \cdot l_i},\ w_{i,max} = \sqrt{S_i \cdot u_i}$$

where S_i denotes the area of module i. This gives us the following ranges for the module width:

$$3.46 \leq w_3 \leq 6.93 \quad (1.1)$$
$$3.83 \leq w_4 \leq 11.07 \quad (1.2)$$

Based on Taylor expansion, we have:

$$h_i = \frac{S_i}{w_{i,max}} + (w_{i,max} - w_i)\frac{S_i}{w_{i,max}^2}$$



Linear Approximation (cont)

Thus, we obtain the following linear approximation for h_3 and h_4:

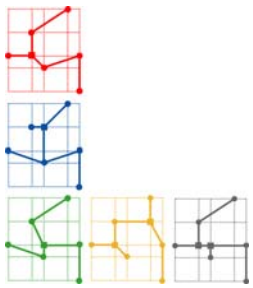
$$h_3 = \frac{24}{\sqrt{24 \cdot 2}} + (\sqrt{24 \cdot 2} - w_3)\frac{24}{24 \cdot 2} = -0.5w_3 + 6.93 \quad (1.3)$$

$$h_4 = \frac{49}{\sqrt{49 \cdot 2.5}} + (\sqrt{49 \cdot 2.5} - w_4)\frac{49}{49 \cdot 2.5} = -0.4w_4 + 8.85 \quad (1.4)$$

Based on Equation (1.1),(1.2), (1.3), and (1.4) we get

$$3.47 \leq h_3 \leq 5.20 \quad (1.5)$$

$$4.42 \leq h_4 \leq 7.32 \quad (1.6)$$



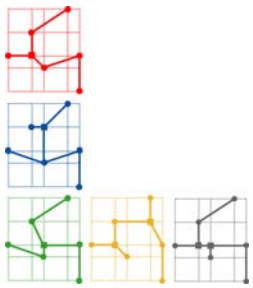
Upper Bound of Chip Dimension

Lastly, we compute the upper bound of chip width $(= W)$ and chip height $(= H)$ as follows:

$$W = \sum w_i = \max\{4, 5\} + \max\{3, 7\} + 6.93 + 11.07 = 30.00$$
$$H = \sum h_i = \max\{4, 5\} + \max\{3, 7\} + 5.20 + 7.32 = 24.52$$

Thus, $M = \max\{W, H\} = 30.00$. We construct the ILP formulation as follows:

Minimize y^*
Subject to

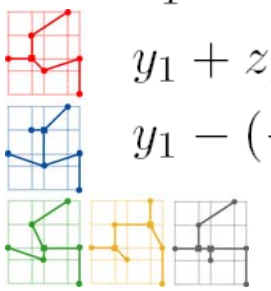


Non-Overlap Constraint

$$x_1 + z_1 h_1 + (1 - z_1) w_1 \leq x_2 + 30.00(x_{12} + y_{12})$$
$$x_1 - z_2 h_2 - (1 - z_2) w_2 \geq x_2 - 30.00(1 - x_{12} + y_{12})$$
$$y_1 + z_1 w_1 + (1 - z_1) h_1 \leq y_2 + 30.00(1 + x_{12} - y_{12})$$
$$y_1 - z_2 w_2 - (1 - z_2) h_2 \geq y_2 - 30.00(2 - x_{12} - y_{12})$$

$$x_1 + z_1 h_1 + (1 - z_1) w_1 \leq x_3 + 30.00(x_{13} + y_{13})$$
$$x_1 - w_3 \geq x_3 - 30.00(1 - x_{13} + y_{13})$$
$$y_1 + z_1 w_1 + (1 - z_1) h_1 \leq y_3 + 30.00(1 + x_{13} - y_{13})$$
$$y_1 - (-0.5 w_3 + 6.93) \geq y_3 - 30.00(2 - x_{13} - y_{13})$$

$$x_1 + z_1 h_1 + (1 - z_1) w_1 \leq x_4 + 30.00(x_{14} + y_{14})$$
$$x_1 - w_4 \geq x_4 - 30.00(1 - x_{14} + y_{14})$$
$$y_1 + z_1 w_1 + (1 - z_1) h_1 \leq y_4 + 30.00(1 + x_{14} - y_{14})$$
$$y_1 - (-0.4 w_4 + 8.85) \geq y_4 - 30.00(2 - x_{14} - y_{14})$$

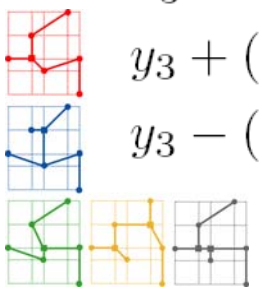


Non-Overlap Constraint (cont)

$$x_2 + z_2 h_2 + (1 - z_2) w_2 \leq x_3 + 30.00(x_{23} + y_{23})$$
$$x_2 - w_3 \geq x_3 - 30.00(1 - x_{23} + y_{23})$$
$$y_2 + z_2 w_2 + (1 - z_2) h_2 \leq y_3 + 30.00(1 + x_{23} - y_{23})$$
$$y_2 - (-0.5 w_3 + 6.93) \geq y_3 - 30.00(2 - x_{23} - y_{23})$$

$$x_2 + z_2 h_2 + (1 - z_2) w_2 \leq x_4 + 30.00(x_{24} + y_{24})$$
$$x_2 - w_4 \geq x_4 - 30.00(1 - x_{24} + y_{24})$$
$$y_2 + z_2 w_2 + (1 - z_2) h_2 \leq y_4 + 30.00(1 + x_{24} - y_{24})$$
$$y_2 - (-0.4 w_4 + 8.85) \geq y_4 - 30.00(2 - x_{24} - y_{24})$$

$$x_3 + w_3 \leq x_4 + 30.00(x_{34} + y_{34})$$
$$x_3 - w_4 \geq x_4 - 30.00(1 - x_{34} + y_{34})$$
$$y_3 + (-0.5 w_3 + 6.93) \leq y_4 + 30.00(1 + x_{34} - y_{34})$$
$$y_3 - (-0.4 w_4 + 8.85) \geq y_4 - 30.00(2 - x_{34} - y_{34})$$



More Constraints

variable type constraints:

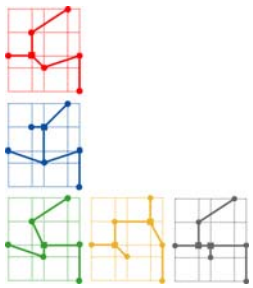
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$$
$$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0,\ y_4 \geq 0$$
$$3.46 \leq w_3 \leq 6.93$$
$$3.83 \leq w_4 \leq 11.07$$
$$x_{12},\ x_{13},\ x_{14},\ x_{23},\ x_{24},\ x_{34} \in \{0, 1\}$$
$$y_{12},\ y_{13},\ y_{14},\ y_{23},\ y_{24},\ y_{34} \in \{0, 1\}$$
$$z_1,\ z_2 \in \{0, 1\}$$

chip width constraints:

$$x_1 + (1 - z_1)w_1 + z_1 h_1 \leq y^*$$
$$x_2 + (1 - z_2)w_2 + z_2 h_2 \leq y^*$$
$$x_3 + w_3 \leq y^*$$
$$x_4 + w_4 \leq y^*$$

chip height constraints:

$$y_1 + (1 - z_1)h_1 + z_1 w_1 \leq y^*$$
$$y_2 + (1 - z_2)h_2 + z_2 w_2 \leq y^*$$
$$y_3 + (-0.5w_3 + 6.93) \leq y^*$$
$$y_4 + (-0.4w_4 + 8.85) \leq y^*$$



Solutions

$y^* = 10.46$

$(x_1, y_1) = (5.46, 5.20, (x_2, y_2) = (0, 0), (x_3, y_3) = (7, 0), (x_4, y_4) = (0, 3)$

$z_1 = 1, z_2 = 1$: (1 and 2 are rotated)

$w_3 = 3.46, \ w_4 = 3.83$

$(x_{12}, y_{12}) = (1, 1) : (1 \text{ is above } 2)$

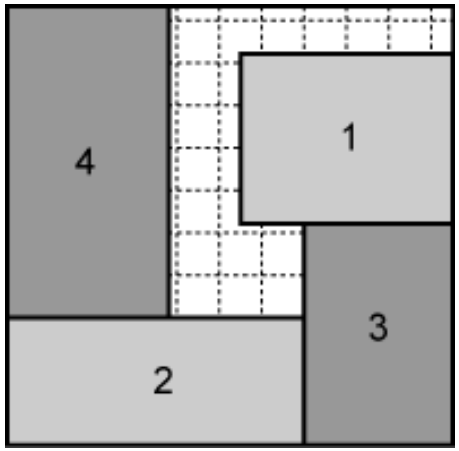
$(x_{13}, y_{13}) = (1, 1) : (1 \text{ is above } 3)$

$(x_{14}, y_{14}) = (1, 0) : (1 \text{ is to the right of } 4)$

$(x_{23}, y_{23}) = (0, 0) : (2 \text{ is to the left of } 3)$

$(x_{24}, y_{24}) = (0, 1) : (2 \text{ is below } 4)$

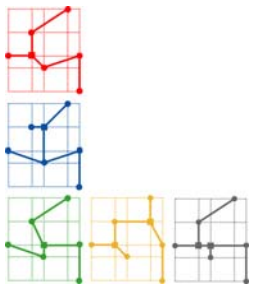
$(x_{34}, y_{34}) = (1, 0) : (3 \text{ is to the right of } 4)$



Based on our linear approximation, i.e., Equation (1.3) and (1.4), we get

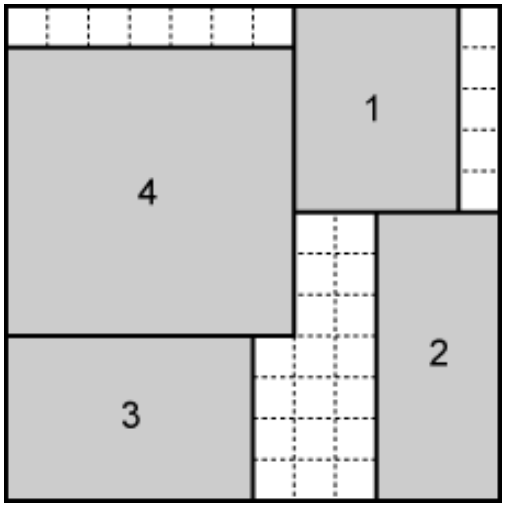
$$h_3 = -0.5w_3 + 6.93 = 5.20$$

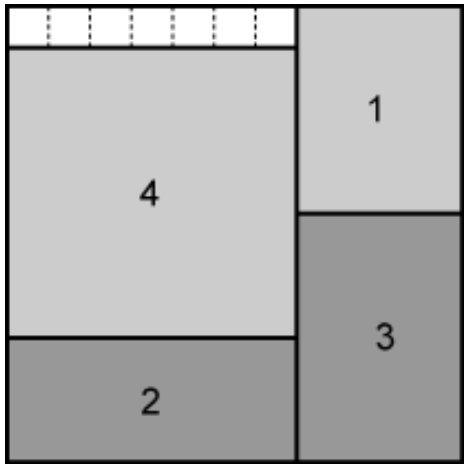
$$h_4 = -0.4w_4 + 8.85 = 7.32$$

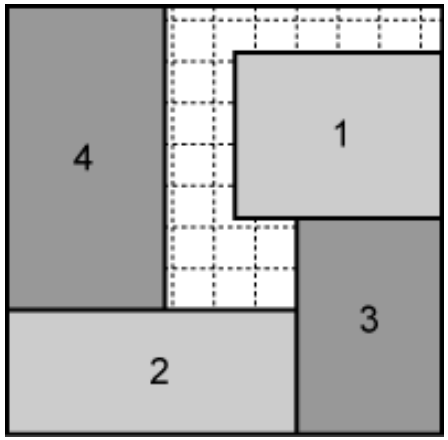


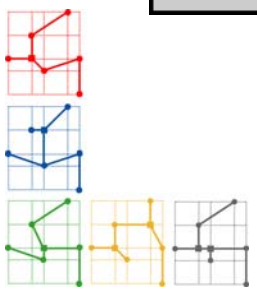
Comparison

- Fixed modules only = 12 × 12
- Rotation allowed = 11 × 11
- Flexible modules used = 10.46 × 10.32









Approximation Error and Overlap

- Due to linear approximation
  - Approximated area of m_3 = 3.46 × 5.2 = 17.99 (actually 24)
  - Approximated area of m_4 = 3.83 × 7.32 = 28.04 (actually 49)
  - Real area of m_3 = 3.46 × 6.94 = 24
  - Real area of m_4 = 3.83 × 12.79 = 49
  - Floorplan area increases, overlap occurs