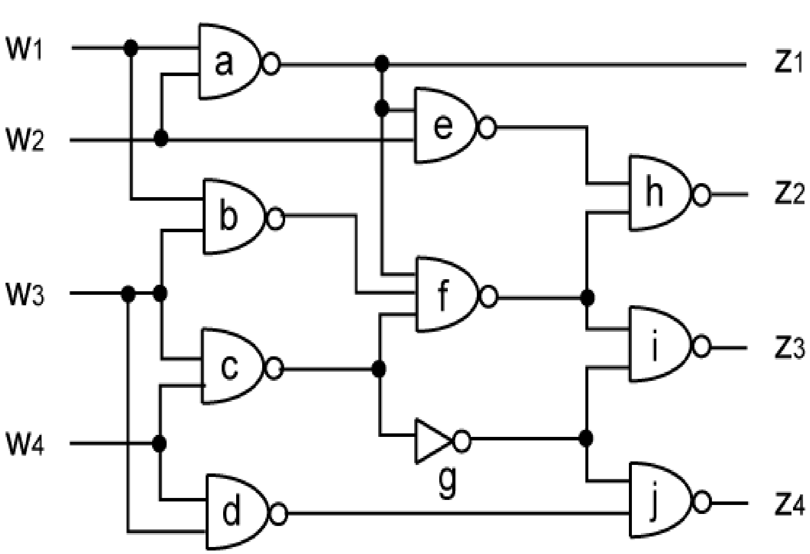
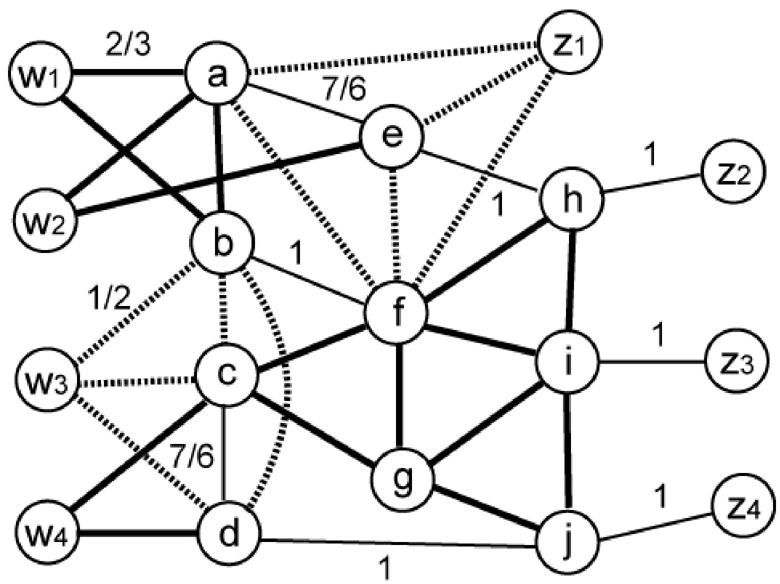
GORDIAN Placement

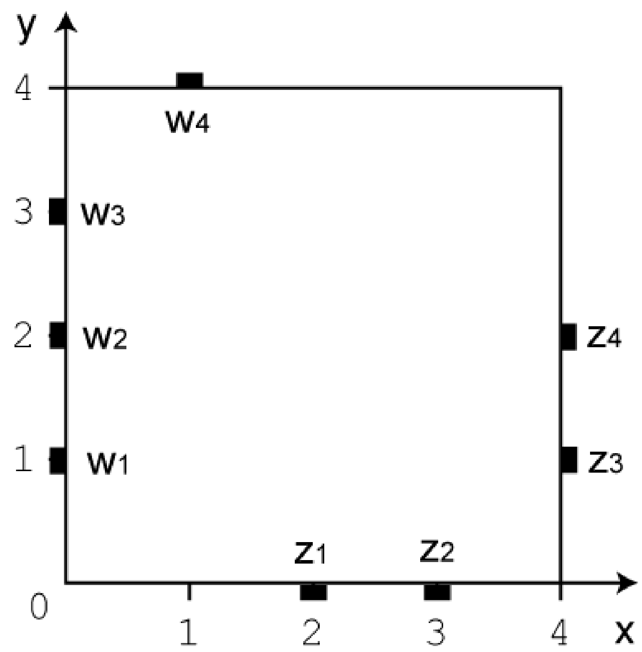
- Perform GORDIAN placement
  - Uniform area and net weight, area balance factor = 0.5
  - Undirected graph model: each edge in *k*-clique gets weight 2/*k*





IO Placement

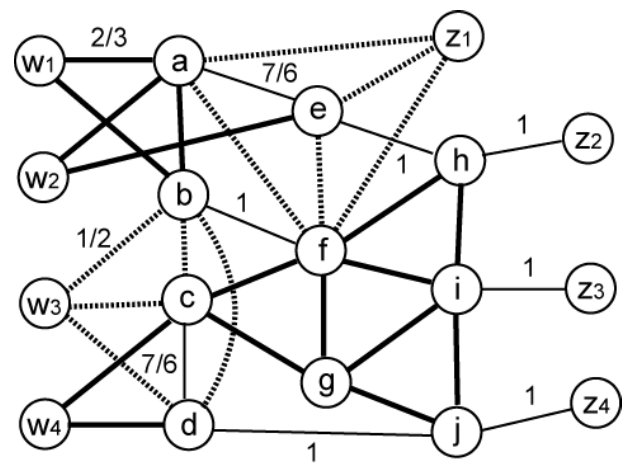
- Necessary for GORDIAN to work



Adjacency Matrix

- Shows connections among movable nodes
  - Among nodes *a* to *j*

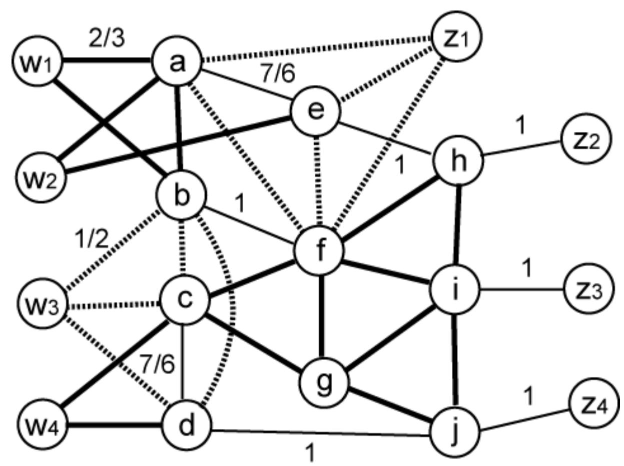
$$\begin{pmatrix}
0 & \frac{2}{3} & 0 & 0 & \frac{7}{6} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
\frac{2}{3} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & \frac{7}{6} & 0 & \frac{2}{3} & \frac{2}{3} & 0 & 0 & 0 \\
0 & \frac{1}{2} & \frac{7}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\frac{7}{6} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 1 & 0 & 0 \\
\frac{1}{2} & 1 & \frac{2}{3} & 0 & \frac{1}{2} & 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\
0 & 0 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & \frac{2}{3} \\
0 & 0 & 0 & 0 & 1 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 & \frac{2}{3} \\
0 & 0 & 0 & 1 & 0 & 0 & \frac{2}{3} & 0 & \frac{2}{3} & 0
\end{pmatrix}$$



Pin Connection Matrix

- Shows connections between movable nodes and IO
  - Rows = movable nodes, columns = IO (fixed)

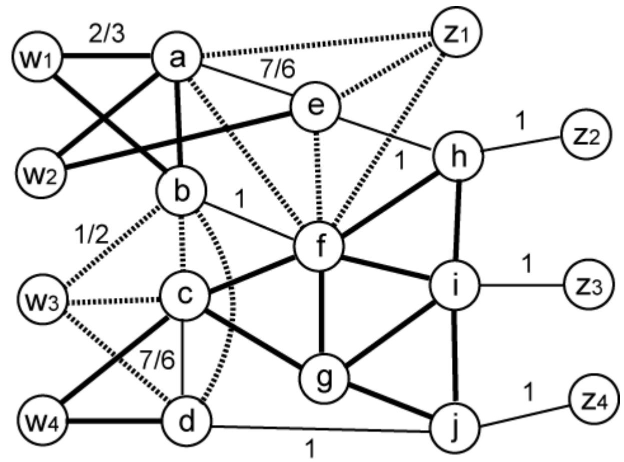
$$
\begin{pmatrix}
\frac{2}{3} & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\
\frac{2}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\
0 & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
$$



Degree Matrix

- Based on both adjacency and pin connection matrices
  - Sum of entries in the same row (= node degree)

$$
\begin{pmatrix}
\frac{25}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{23}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{25}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{23}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{23}{6} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{31}{6} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{10}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{10}{3}
\end{pmatrix}
$$



Laplacian Matrix

- Degree matrix minus adjacency matrix

$$
\begin{pmatrix}
\frac{25}{6} & -\frac{2}{3} & 0 & 0 & -\frac{7}{6} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
-\frac{2}{3} & \frac{23}{6} & -\frac{1}{2} & -\frac{1}{2} & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & \frac{25}{6} & -\frac{7}{6} & 0 & -\frac{2}{3} & -\frac{2}{3} & 0 & 0 & 0 \\
0 & -\frac{1}{2} & -\frac{7}{6} & \frac{23}{6} & 0 & 0 & 0 & 0 & 0 & -1 \\
-\frac{7}{6} & 0 & 0 & 0 & \frac{23}{6} & -\frac{1}{2} & 0 & -1 & 0 & 0 \\
-\frac{1}{2} & -1 & -\frac{2}{3} & 0 & -\frac{1}{2} & \frac{31}{6} & -\frac{2}{3} & -\frac{2}{3} & -\frac{2}{3} & 0 \\
0 & 0 & -\frac{2}{3} & 0 & 0 & -\frac{2}{3} & \frac{8}{3} & 0 & -\frac{2}{3} & -\frac{2}{3} \\
0 & 0 & 0 & 0 & -1 & -\frac{2}{3} & 0 & \frac{10}{3} & -\frac{2}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{2}{3} & -\frac{2}{3} & -\frac{2}{3} & \frac{11}{3} & -\frac{2}{3} \\
0 & 0 & 0 & -1 & 0 & 0 & -\frac{2}{3} & 0 & -\frac{2}{3} & \frac{10}{3}
\end{pmatrix}
$$

Fixed Pin Vectors

- Based on pin connection matrix and IO location

  Each entry i in d_x, denoted $d_{x,i}$, is computed as follows:

  $$d_{x,i} = -\sum_j p_{ij} \cdot x(p_j)$$

  where p_{ij} denotes the entry of the pin connection matrix, and $x(p_j)$ is the x-coordinate of the corresponding IO pin j.

  - Y-direction is defined similarly

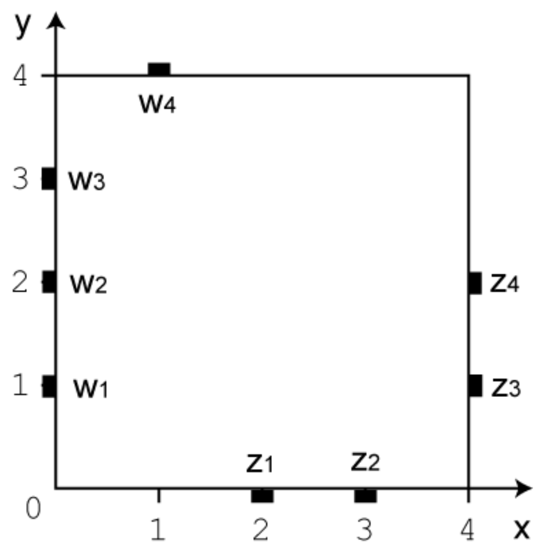
Fixed Pin Vectors (cont)

$$d_{x,1} = -(\frac{2}{3}\cdot 0 + \frac{2}{3}\cdot 0 + 0\cdot 0 + 0\cdot 1 + \frac{1}{2}\cdot 2 + 0\cdot 3 + 0\cdot 4 + 0\cdot 4) = -1$$

By examining the remaining 9 movable cells, we get

$$d_x^T = \begin{pmatrix} -1 & 0 & -\frac{2}{3} & -\frac{2}{3} & -1 & -1 & 0 & -3 & -4 & -4 \end{pmatrix}$$

$$\begin{pmatrix} \frac{2}{3} & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



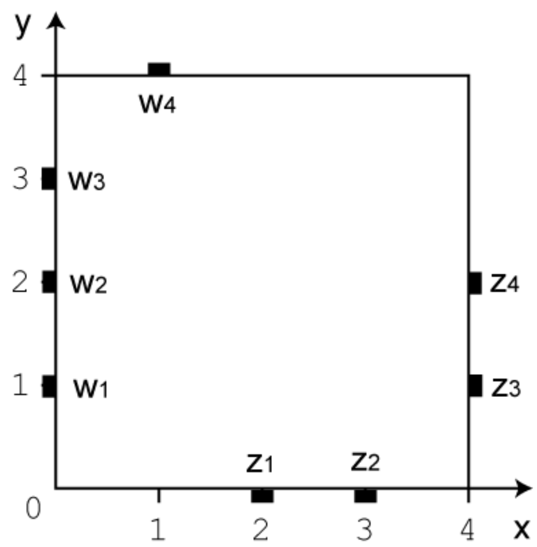
Fixed Pin Vectors (cont)

$$d_{y,1} = -(\frac{2}{3}\cdot 1 + \frac{2}{3}\cdot 2 + 0\cdot 3 + 0\cdot 4 + \frac{1}{2}\cdot 0 + 0\cdot 0 + 0\cdot 1 + 0\cdot 2) = -2$$

By examining the remaining 9 movable cells, we get

$$d_y^T = \begin{pmatrix} -2 & -\frac{13}{6} & -\frac{25}{6} & -\frac{25}{6} & -\frac{4}{3} & 0 & 0 & 0 & -1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} \frac{2}{3} & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



Level 0 QP Formulation

- No constraint necessary

Minimize

$$\phi(x) = \frac{1}{2} x^T C x + {d_x}^T x$$

and

$$\phi(y) = \frac{1}{2} y^T C y + {d_y}^T y$$

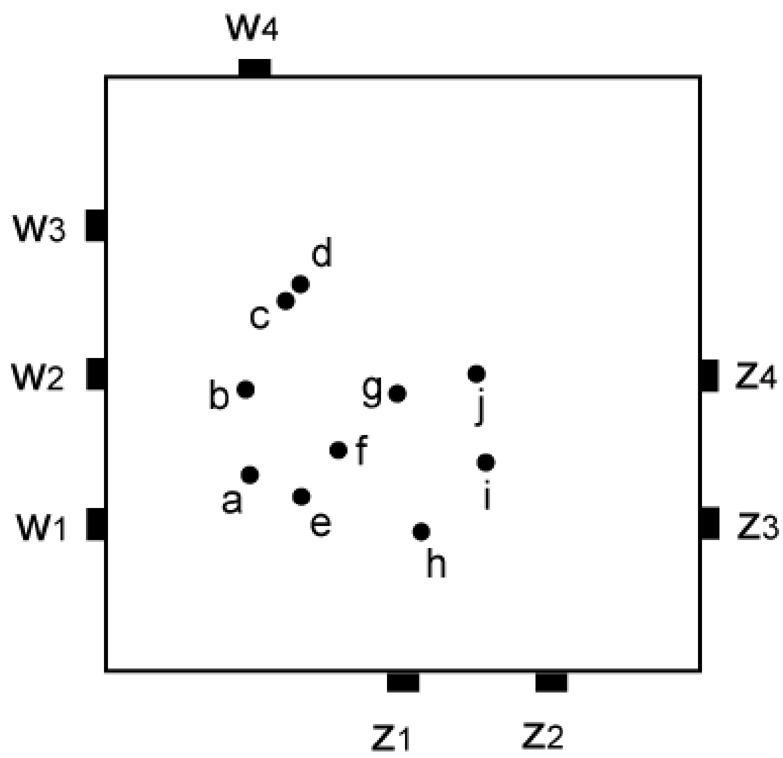
We use MOSEK and obtain the following solution:

$$x^T = \begin{pmatrix} 0.95 & 0.92 & 1.21 & 1.32 & 1.32 & 1.61 & 1.98 & 2.13 & 2.59 & 2.51 \end{pmatrix}$$

$$y^T = \begin{pmatrix} 1.27 & 1.83 & 2.48 & 2.61 & 1.16 & 1.45 & 1.84 & 0.92 & 1.41 & 2.03 \end{pmatrix}$$

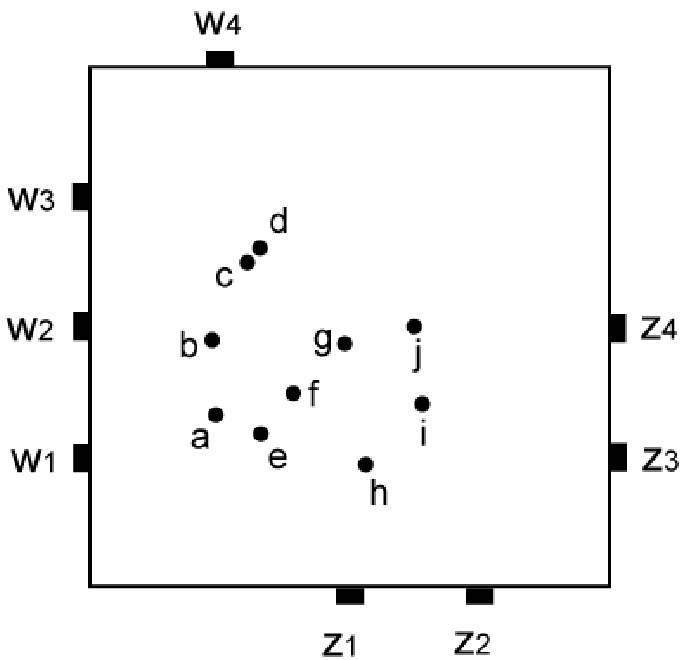
Level 0 Placement

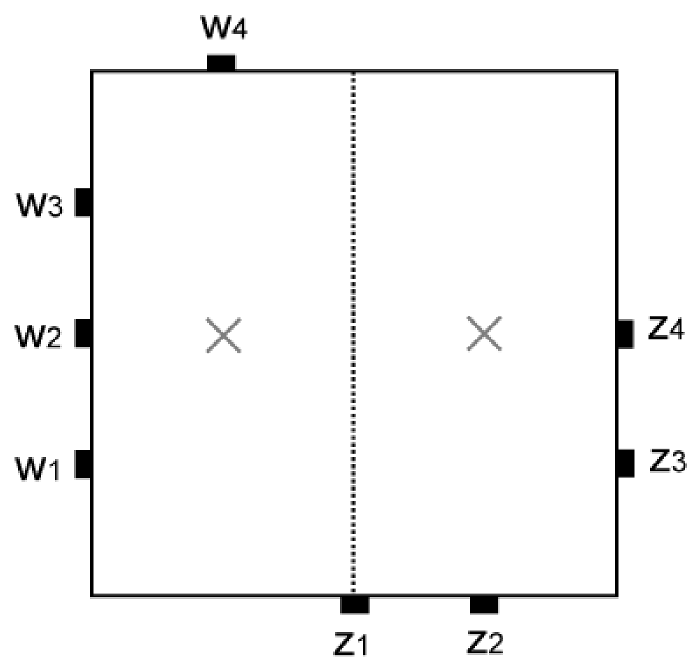
- Cells with real dimension will overlap



Level 1 Partitioning

- Perform level 1 partitioning
  - Obtain center locations for center-of-gravity constraints





Level 1 Constraint

We first sort the nodes based on their x-coordinates:

$$\{b, a, c, e, d, f, g, h, j, i\}$$

We perform partitioning under $\alpha = 0.5$:

$$S_{\rho'} = \{b, a, c, e, d\},\ S_{\rho''} = \{f, g, h, j, i\}$$

The center location vectors are:

$$u_x^{(1)} = \begin{pmatrix} 1 \\ 3 \end{pmatrix},\ u_y^{(1)} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

We build the matrix $A^{(1)}$ for the center-of-gravity constraint at level $l = 1$:

$$A^{(1)} = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

Level 1 LQP Formulation

We now solve the following Linearly constrained QP (LQP) to obtain the new placement for the movable nodes:

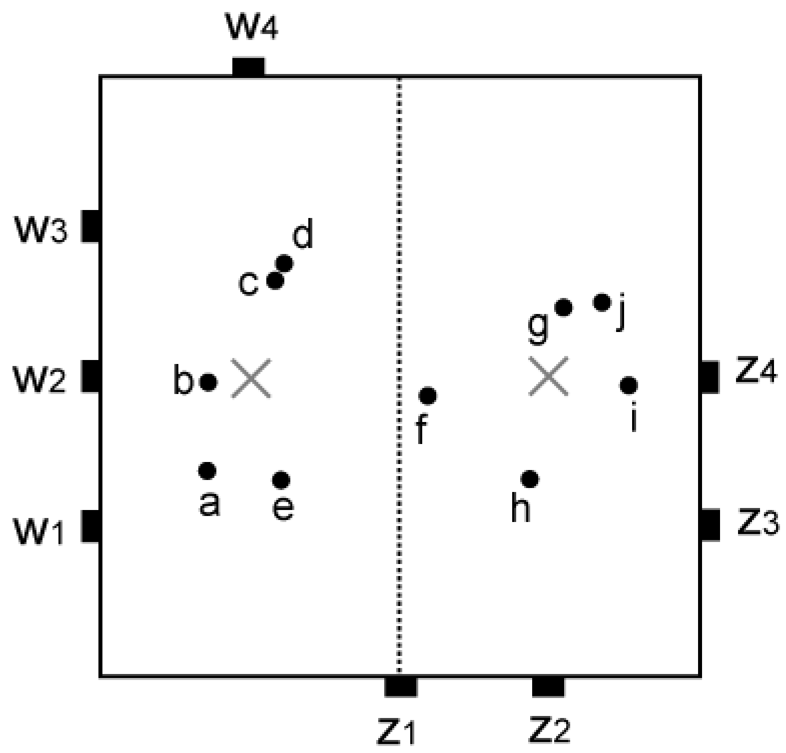
$$\text{Minimize } \phi(x) = \frac{1}{2}x^T C x + {d_x}^T x, \text{ subject to } A^{(1)} \cdot x = u_x^{(1)}$$

$$\text{Minimize } \phi(y) = \frac{1}{2}y^T C y + {d_y}^T y, \text{ subject to } A^{(1)} \cdot y = u_y^{(1)}$$

The solutions are as follows:

$$x^T = \begin{pmatrix} 0.70 & 0.71 & 1.17 & 1.21 & 1.22 & 2.17 & 3.10 & 2.84 & 3.56 & 3.33 \end{pmatrix}$$
$$y^T = \begin{pmatrix} 1.34 & 1.94 & 2.66 & 2.76 & 1.30 & 1.83 & 2.45 & 1.32 & 1.91 & 2.49 \end{pmatrix}$$

Level 1 Placement



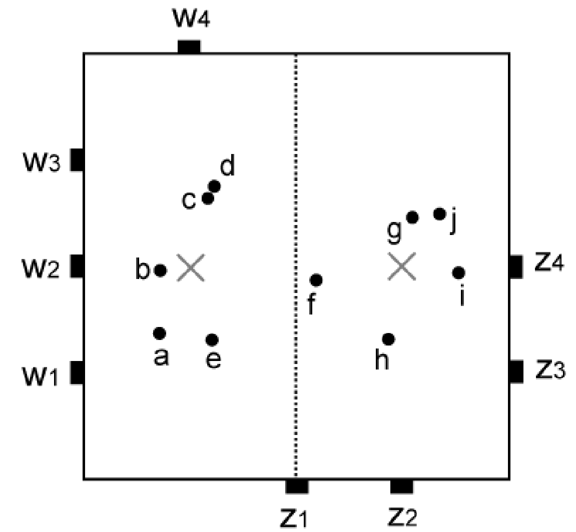
Verification

- Verify that the constraints are satisfied in the left partition

The following cells are partitioned to the left: $a(0.70, 1.34)$, $b(0.71, 1.94)$, $c(1.17, 2.66)$, $d(1.21, 2.76)$, and $e(1.22, 1.30)$. Thus, the center of gravity is located at:

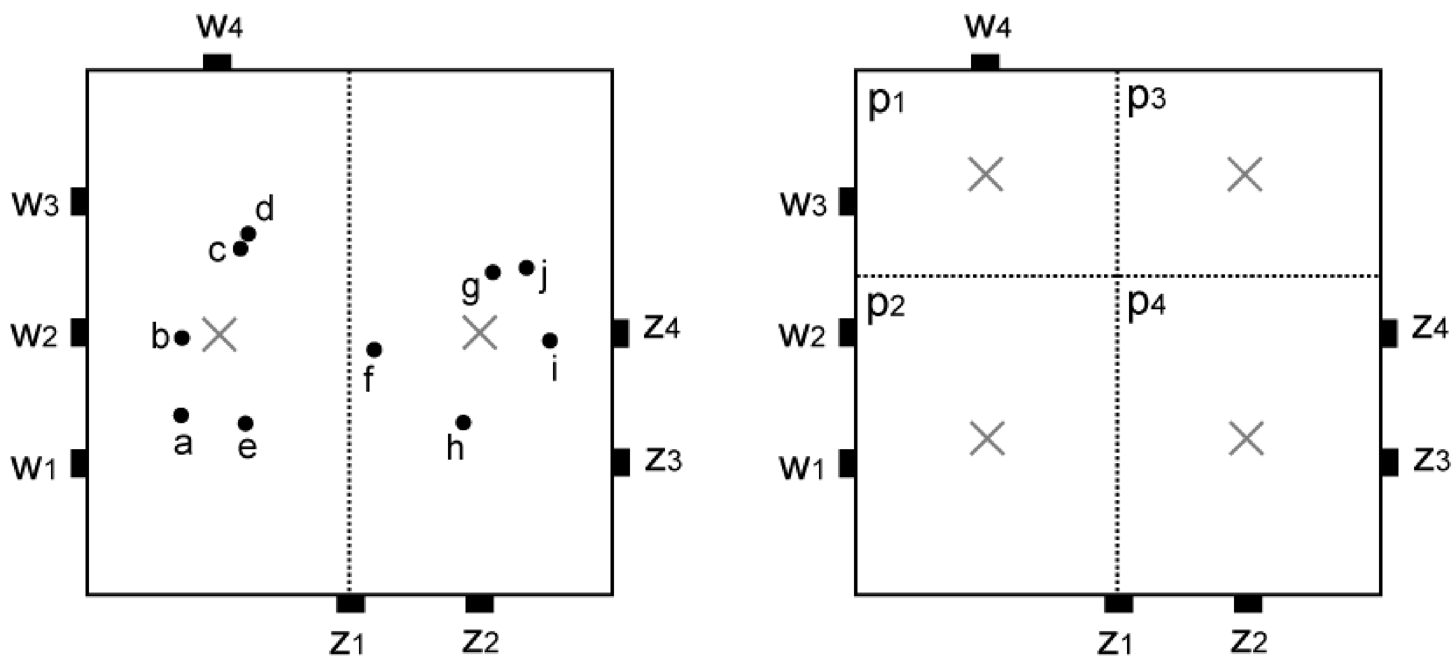
$$\frac{0.70 + 0.71 + 1.17 + 1.21 + 1.22}{5} = 1.00$$
$$\frac{1.34 + 1.94 + 2.66 + 2.76 + 1.30}{5} = 2.00$$

This agrees with the center location $(1, 2)$.



Level 2 Partitioning

- Add two more cut-lines
  - This results in $p_1=\{c,d\}$, $p_2=\{a,b,e\}$, $p_3=\{g,j\}$, $p_4=\{f,h,i\}$



Level 2 Constraint

The center location vectors are:

$$u_x^{(2)} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 3 \end{pmatrix}, \quad u_y^{(2)} = \begin{pmatrix} 3.2 \\ 1.2 \\ 3.2 \\ 1.2 \end{pmatrix}$$

Next, we build the matrix $A^{(2)}$ for the center-of-gravity constraint at level $l = 2$. Recall that $p_1 = \{c, d\}$, $p_2 = \{a, b, e\}$, $p_3 = \{g, j\}$, $p_4 = \{f, h, i\}$. Thus,

$$A^{(2)} = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

where the rows denote the partitions p_1 through p_4, and the columns denote the cells a through j.

Level 2 LQP Formulation

We now solve the following LQP to obtain the placement of the movable nodes:

$$\text{Minimize } \phi(x) = \frac{1}{2}x^T Cx + {d_x}^T x, \text{ subject to } A^{(2)} \cdot x = u_x^{(2)}$$

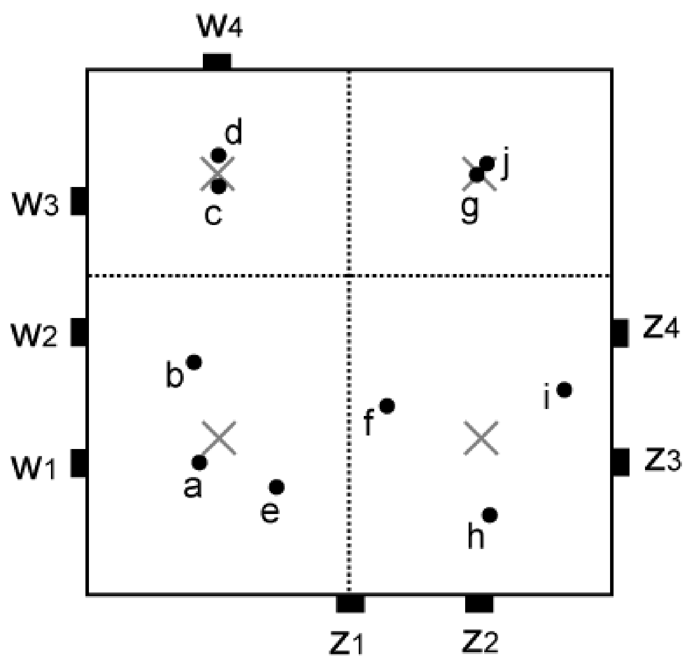
$$\text{Minimize } \phi(y) = \frac{1}{2}y^T Cy + {d_y}^T y, \text{ subject to } A^{(2)} \cdot y = u_y^{(2)}$$

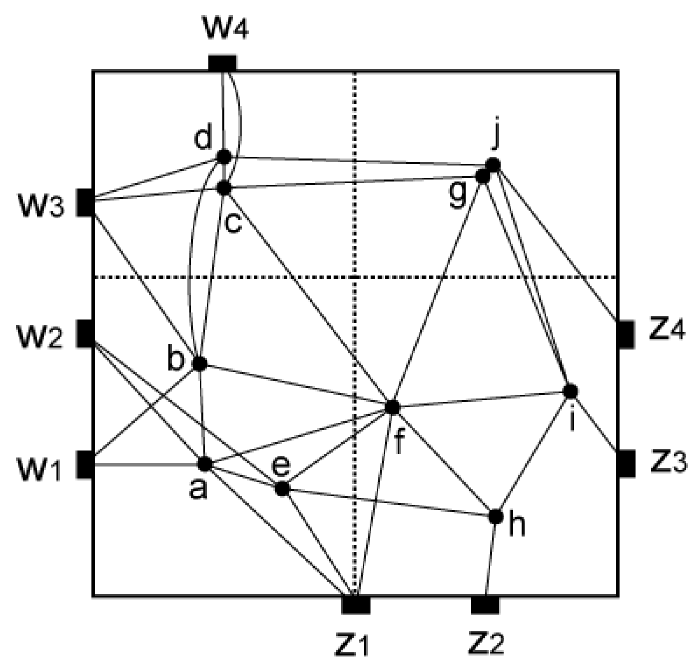
The solutions are as follows:

$$x^T = \begin{pmatrix} 0.83 & 0.78 & 1.00 & 1.00 & 1.39 & 2.28 & 2.89 & 3.06 & 3.66 & 3.11 \end{pmatrix}$$
$$y^T = \begin{pmatrix} 1.01 & 1.78 & 3.08 & 3.32 & 0.82 & 1.44 & 3.18 & 0.59 & 1.57 & 3.22 \end{pmatrix}$$

Level 2 Placement

- Clique-based wiring is shown





Summary

- Center-of-gravity constraint
  - Helps spread the cells evenly while monitoring wirelength
  - Removes overlaps among the cells (with real dimension)