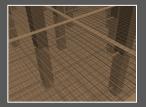
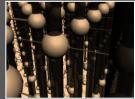
# **ECE 6133 Project Overview**

Spring 2025





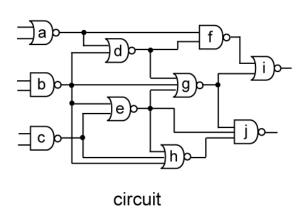




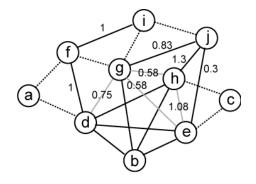
Sung Kyu Lim
limsk@ece.gatech.edu
Georgia Institute of Technology

## 1/5. EIG Partitioning

### Circuit partitioning using matrices and eigenvectors

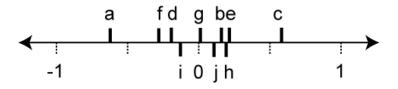


	a	b	c	d	e	f	g	h	i	j
$\overline{a}$	1.0	0	0	-0.5	0	-0.5	0	0	0	0
b	0	1.0	0	-0.25	-0.25	0	-0.25	-0.25	0	0
c	0	0	1.0	0	-0.5	0	0	-0.5	0	0
d	-0.5	-0.25	0	3.0	-0.25	-1.0	-0.75	-0.25	0	0
e	0	-0.25	-0.5	-0.25	2.99	0	-0.58	-1.08	0	-0.33
f	-0.5	0	0	-1.0	0	3.0	-0.5	0	-1.0	0
g	0	-0.25	0	-0.75	-0.58	-0.5	3.99	-0.58	-0.5	-0.83
h	0	-0.25	-0.5	-0.25	-1.08	0	-0.58	3.99	0	-1.33
i	0	0	0	0	0	-1.0	-0.5	0	2.0	-0.5
j	0	0	0	0	-0.33	0	-0.83	-1.33	-0.5	2.99



clique-based model

### Laplacian matrix L



2nd smallest eigenvector of L gives linear placement

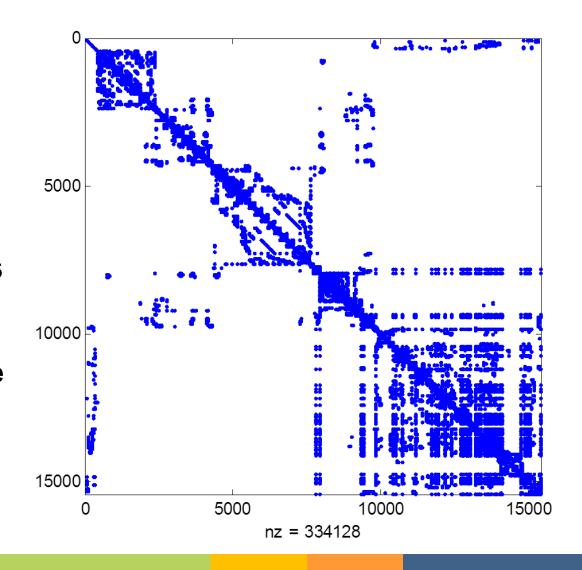
# Laplacian Sparsity: Small Circuit

## industry3.hgr

- 15,406 nodes
- 21,923 nets
- 237M entries in matrix
- 334k nonzero entries
- 0.14% fill factor

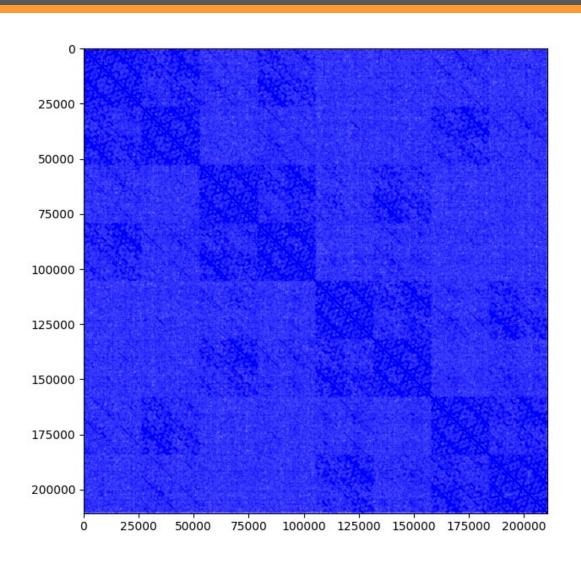
### Sparse matrix methods

 enables us to consider systems 700X larger than otherwise possible

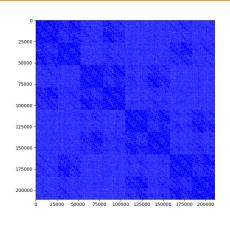


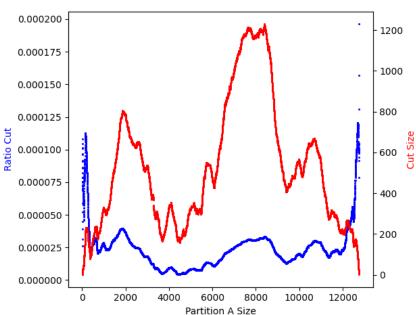
# Laplacian Sparsity: Large Circuit

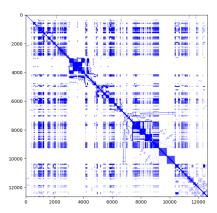
- IBM18 benchmark
  - 210,613 cells
  - Laplacian matrix fill rate:0.01%!
  - 178 GB of memory required to store in a normal matrix
  - Sparse matrix can bring that down to 2.5 MB

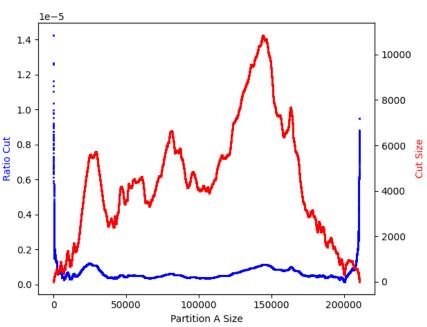


# Cutsize vs. Ratio Cut Landscape





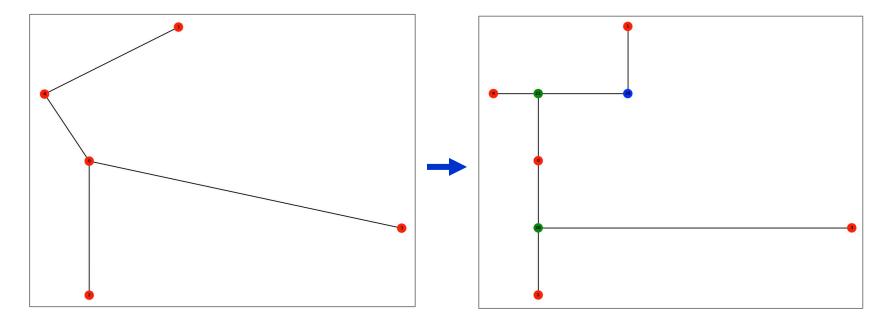




# 4/5. L-Shaped vs. Borah Routing

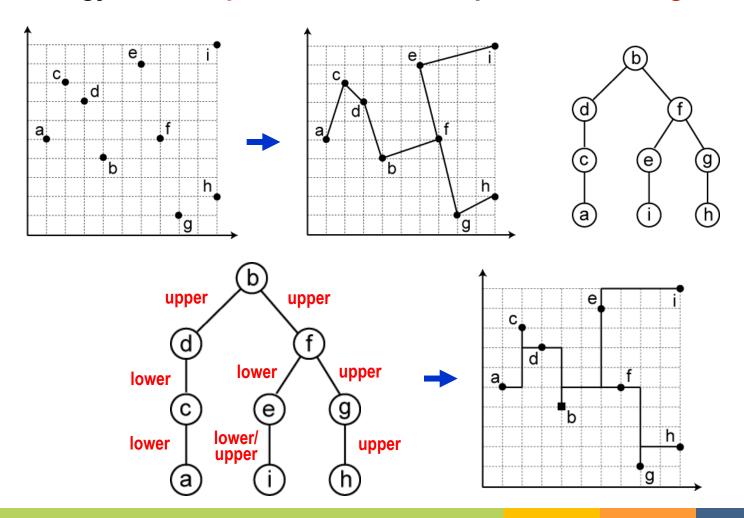
## Steiner points

- Reduce the overall wirelength
- Finding Steiner points is NP-hard....
- So, we rely on heuristics



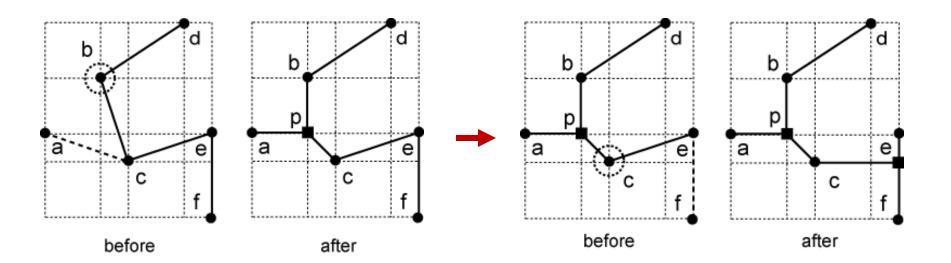
## L-Shaped Steiner Routing

Strategy: build separable MST and improve with L-edges



# **Borah Routing**

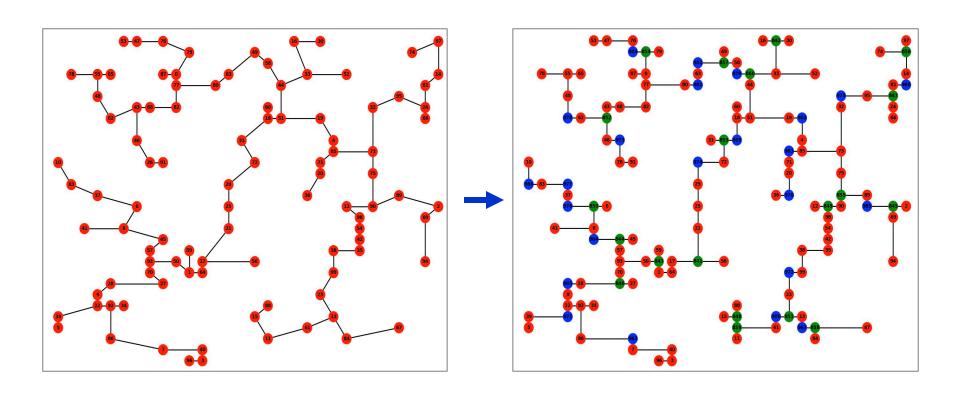
Based on node/edge pairing



WL reduces from 20 to 18

WL reduces from 18 to 17

# **Borah Routing Sample**

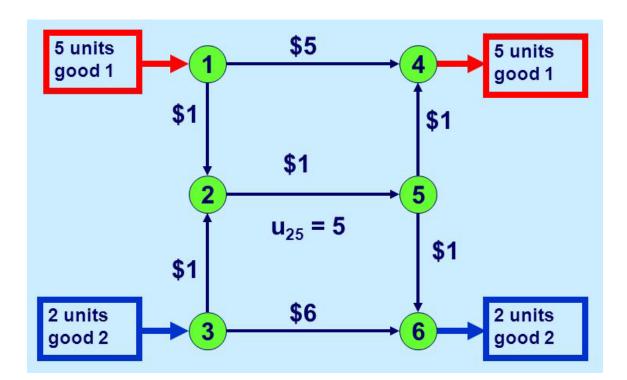


**initial (WL = 242)** 

final (WL = 221), 20 Steiner pt added Took 1.2 sec

# 5/5. Multi-Commodity Flow Routing

- Cost minimization problem
  - How do we ship the units so that the overall cost is minimized?
  - Assume the capacity of each edge is 5 units

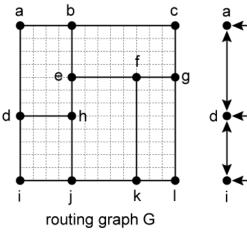


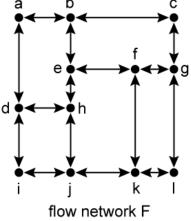
## MCF-based Multi-net Routing

## Set up ILP for MCF routing

- Capacity of each edge in G is 2
- Each edge in G becomes a pair of bi-directional arcs in F

$$- n_1 = \{a,l\}, n_2 = \{i,c\}, n_3 = \{d,f\}, n_4 = \{k,d\}, n_5 = \{g,h\}, n_6 = \{b,k\}$$





arc	cost	arc	cost	arc	cost	arc	cost
$\overline{(a,b)}$	4	(b,a)	4	(b,c)	8	(c,b)	8
(d,h)	4	(h,d)	4	(e,f)	5	(f,e)	5
(f,g)	3	(g,f)	3	(i,j)	4	(j,i)	4
(j,k)	5	(k, j)	5	(k,l)	3	(l,k)	3
(a,d)	7	(d,a)	7	(d,i)	5	(i,d)	5
(b, e)	4	(e,b)	4	(e,h)	3	(h, e)	3
(h, j)	5	(j,h)	5	(f,k)	8	(k, f)	8
(c,g)	4	(g,c)	4	(g, l)	8	(l,g)	8

## **ILP Formulation**

$$\begin{split} &4(x_{a,b}^1+\dots+x_{a,b}^6)+4(x_{b,a}^1+\dots+x_{b,a}^6)+8(x_{b,c}^1+\dots+x_{b,c}^6)+\\ &8(x_{c,b}^1+\dots+x_{c,b}^6)+4(x_{d,h}^1+\dots+x_{d,h}^6)+4(x_{h,d}^1+\dots+x_{h,d}^6)+\\ &5(x_{e,f}^1+\dots+x_{e,f}^6)+5(x_{f,e}^1+\dots+x_{f,e}^6)+3(x_{f,g}^1+\dots+x_{f,g}^6)+\\ &3(x_{g,f}^1+\dots+x_{g,f}^6)+4(x_{i,j}^1+\dots+x_{i,j}^6)+4(x_{j,i}^1+\dots+x_{j,i}^6)+\\ &5(x_{j,k}^1+\dots+x_{j,k}^6)+5(x_{k,j}^1+\dots+x_{k,j}^6)+3(x_{k,l}^1+\dots+x_{k,l}^6)+\\ &3(x_{l,k}^1+\dots+x_{l,k}^6)+7(x_{a,d}^1+\dots+x_{a,d}^6)+7(x_{d,a}^1+\dots+x_{d,a}^6)+\\ &5(x_{d,i}^1+\dots+x_{d,i}^6)+5(x_{i,d}^1+\dots+x_{i,d}^6)+4(x_{b,e}^1+\dots+x_{b,e}^6)+\\ &4(x_{e,b}^1+\dots+x_{e,b}^6)+3(x_{e,h}^1+\dots+x_{e,h}^6)+3(x_{h,e}^1+\dots+x_{h,e}^6)+\\ &5(x_{h,j}^1+\dots+x_{h,j}^6)+5(x_{j,h}^1+\dots+x_{j,h}^6)+8(x_{f,k}^1+\dots+x_{f,k}^6)+\\ &8(x_{k,f}^1+\dots+x_{k,f}^6)+8(x_{l,g}^1+\dots+x_{l,g}^6)+\\ &8(x_{g,l}^1+\dots+x_{g,l}^6)+8(x_{l,g}^1+\dots+x_{l,g}^6)+\\ \end{split}$$

#### objective function

$$\begin{aligned} x_{a,b}^1 + x_{a,d}^1 - x_{b,a}^1 - x_{d,a}^1 &= 1 \\ x_{a,b}^2 + x_{a,d}^2 - x_{b,a}^2 - x_{d,a}^2 &= 0 \\ x_{a,b}^3 + x_{a,d}^3 - x_{b,a}^3 - x_{d,a}^3 &= 0 \\ x_{a,b}^4 + x_{a,d}^4 - x_{b,a}^4 - x_{d,a}^4 &= 0 \\ x_{a,b}^5 + x_{a,d}^5 - x_{b,a}^5 - x_{d,a}^5 &= 0 \\ x_{a,b}^6 + x_{a,d}^6 - x_{b,a}^6 - x_{d,a}^6 &= 0 \end{aligned}$$

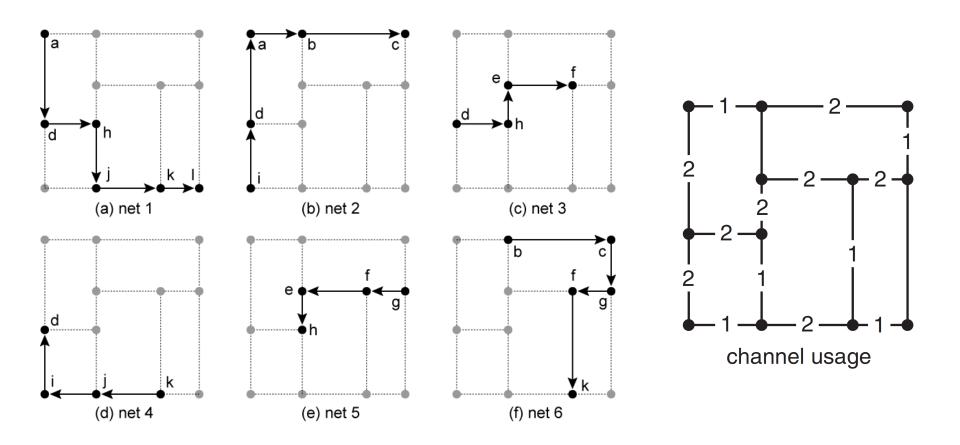
#### demand constraints

$$\begin{split} x_{a,b}^1 + \cdots x_{a,b}^6 + x_{b,a}^1 + \cdots x_{b,a}^6 &\leq 2 \\ x_{b,c}^1 + \cdots + x_{b,c}^6 + x_{c,b}^1 + \cdots + x_{c,b}^6 &\leq 2 \\ x_{d,h}^1 + \cdots + x_{d,h}^6 + x_{h,d}^1 + \cdots + x_{h,d}^6 &\leq 2 \\ x_{e,f}^1 + \cdots + x_{e,f}^6 + x_{f,e}^1 + \cdots + x_{f,e}^6 &\leq 2 \\ \cdots \\ x_{h,j}^1 + \cdots + x_{h,j}^6 + x_{j,h}^1 + \cdots + x_{j,h}^6 &\leq 2 \\ x_{f,k}^1 + \cdots + x_{f,k}^6 + x_{k,f}^1 + \cdots + x_{k,f}^6 &\leq 2 \\ x_{c,g}^1 + \cdots + x_{c,g}^6 + x_{g,c}^1 + \cdots + x_{l,g}^6 &\leq 2 \\ x_{g,l}^1 + \cdots + x_{g,l}^6 + x_{l,g}^1 + \cdots + x_{l,g}^6 &\leq 2 \end{split}$$

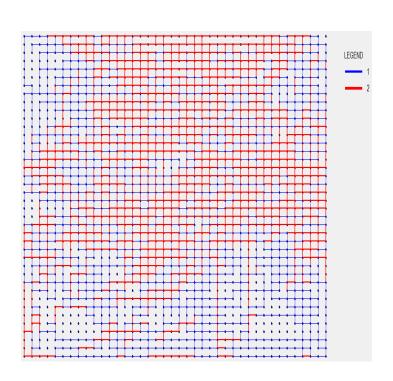
#### capacity constraints

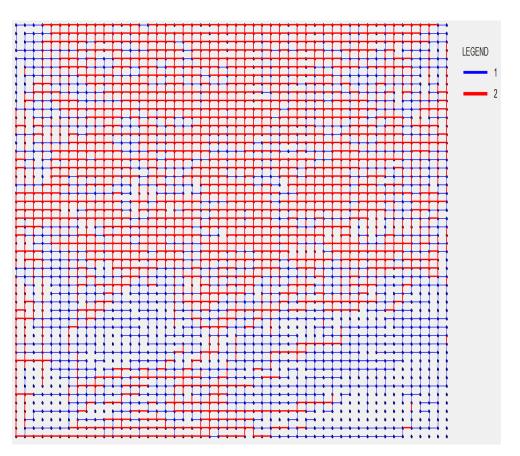
# **ILP-based Routing Solution**

## Using off-the-shelf ILP solver



# Sample Routing Results





200 net benchmark usage 1:1298 edges

usage 2: 1412 edges

350 net benchmark usage 1:1849 edges usage 2:2468 edges